Chapter 11

Three-Dimensional Figures

What You'll Learn

- **Lesson 11-1** Identify three-dimensional figures.
- **Lessons 11-2 and 11-3** Find volumes of prisms, cylinders, pyramids, and cones.
- **Lessons 11-4 and 11-5** Find surface areas of prisms, cylinders, pyramids, and cones.
- **Lesson 11-6** Identify similar solids.
- **Lesson 11-7** Use precision and significant digits to describe measurements.

Key Vocabulary

- polyhedron (p. 556)
- volume (p. 563)
- surface area (p. 573)
- similar solids (p. 584)
- precision (p. 590)

Why It's Important

Three-dimensional figures have special characteristics. These characteristics are important when architects are designing buildings and other three-dimensional structures.

You will investigate the characteristics of architectural structures in Lesson 11-1.

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 11.

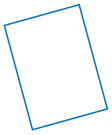
For Lesson 11-1 — Polygons

Determine whether each figure is a polygon. If it is, classify the polygon.
(For review, see Lesson 10-6.)

1. 

2. 

3. 

4. 

For Lessons 11-2 through 11-5 — Multiplying Rational Numbers

Find each product. *(For review, see Lesson 5-3.)*

5. $8.5 \cdot 2$

6. $3.2(3.2)10$

7. $\frac{1}{2} \cdot 14$

8. $\frac{1}{2}(6.4)(5)$

9. $\frac{1}{3}(50)(9.3)$

10. $\frac{1}{3}\left(\frac{1}{2} \cdot 3 \cdot 8\right)$

For Lesson 11-6 — Proportions

Determine whether each pair of ratios forms a proportion. *(For review, see Lesson 6-2.)*

11. $\frac{3}{8}, \frac{9}{24}$

12. $\frac{7}{2}, \frac{14}{6}$

13. $\frac{18}{32}, \frac{9}{16}$

14. $\frac{12}{15}, \frac{4}{5}$

15. $\frac{1.2}{5}, \frac{6}{25}$

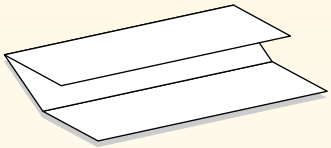
16. $\frac{1.6}{2}, \frac{3.6}{6}$

FOLDABLES™ Study Organizer

Surface Area and Volume Make this Foldable to help you organize your notes. Begin with a plain piece of 11" × 17" paper.

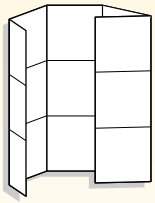
Step 1 Fold

Fold the paper in thirds lengthwise.



Step 2 Open and Fold

Fold a 2" tab along the short side. Then fold the rest in fourths.



Step 3 Label

Draw lines along folds and label as shown.

Ch. 11	Prisms	Cylinders	Pyramids	Cones
Volume				
Surface Area				

Reading and Writing As you read and study the chapter, write the formulas for surface area and volume and list the characteristics of each three-dimensional figure.

Geometry Activity

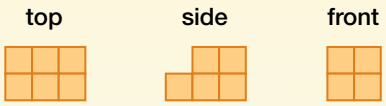
A Preview of Lesson 11-1

California Standards Standard 7MG3.5 Construct two-dimensional patterns for three-dimensional models, such as cylinders, prisms, and cones.

Building Three-Dimensional Figures

Activity 1

Different views of a stack of cubes are shown at the right. A point of view is called a **perspective**. You can build or draw a three-dimensional figure using different perspectives.

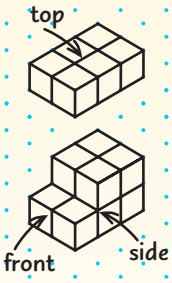


When drawing figures, use isometric dot paper.

Step 1 Use the top view to build the base of the figure. The top view shows that the base is a 2-by-3 rectangle.

Step 2 Use the side view to complete the figure. The side view shows that the height of the first row is 1 unit, and the height of the second and third rows is 2 units.

Step 3 Use the front view to check the figure. The front view is a 2-by-2 square. This shows that the overall height and width of the figure is 2 units. So, the figure is correct.

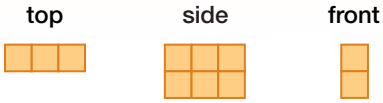


Model

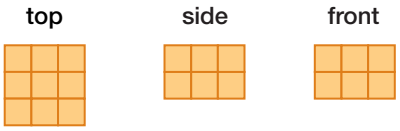
The top view, a side view, and the front view of three-dimensional figures are shown. Use cubes to build each figure. Draw your model on isometric dot paper.

1.

2.



3.



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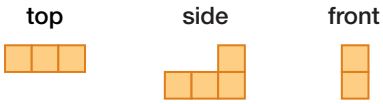
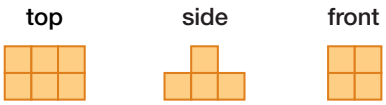
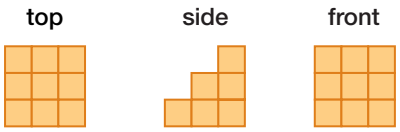


5.



6.



Draw and label the top view, a side view, and the front view for each figure.

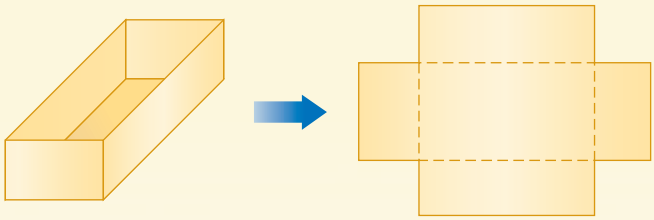
7.

8.

9.

Activity 2

Suppose you cut a cardboard box along its edges, open it up, and lay it flat. The result is a two-dimensional figure called a net. **Nets** are two-dimensional patterns for three-dimensional figures.



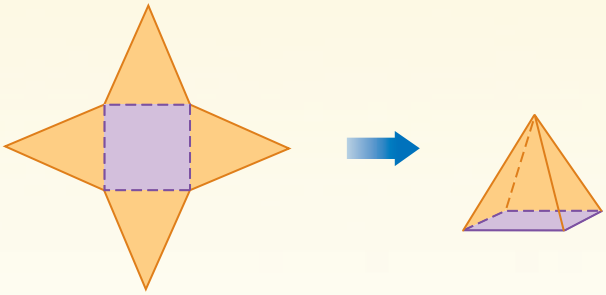
Nets can help you see the regions or *faces* that make up the surface of a figure. So, you can use a net to build a three-dimensional figure.

Step 1 Copy the net on a piece of paper, shading the base as shown.

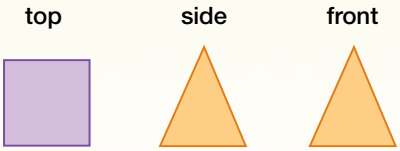
Step 2 Use scissors to cut out the net.

Step 3 Fold on the dashed lines.

Step 4 Tape the sides together.



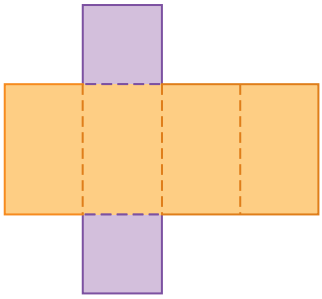
Different views of this figure are shown.



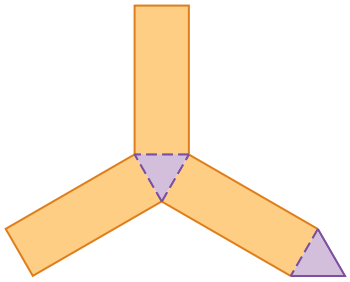
Model

Copy each net. Then cut out the net and fold on the dashed lines to make a 3-dimensional figure, using the purple areas as the bases. Sketch each figure, and draw and label the top view, a side view, and the front view.

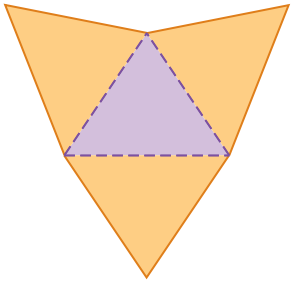
10.



11.



12.



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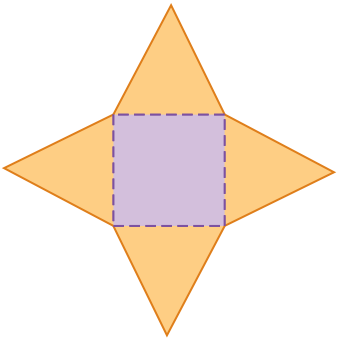

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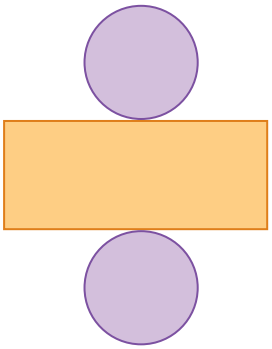

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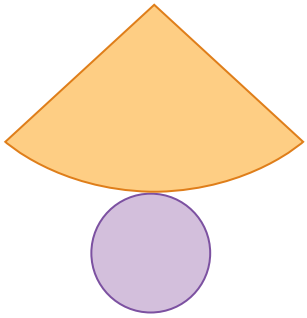


11-1 Three-Dimensional Figures

California Standards Standard 7MG3.6 Identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect). (Key)

What You'll Learn

- Identify three-dimensional figures.
- Identify diagonals and skew lines.

Vocabulary

- plane
- solid
- polyhedron
- edge
- vertex
- face
- prism
- base
- pyramid
- skew lines

How are 2-dimensional figures related to 3-dimensional figures?

Great Pyramid, Egypt

Inner Harbor & Trade Center, Baltimore

a. If you observed the Great Pyramid or the Inner Harbor & Trade Center from directly above, what geometric figure would you see?

b. If you stood directly in front of each structure, what geometric figure would you see?

c. Explain how you can see different polygons when looking at a 3-dimensional figure.

Study Tip

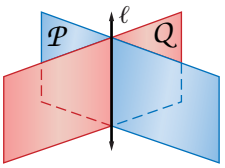
Dimensions

A two-dimensional figure has two dimensions, length and width. A three-dimensional figure has three dimensions, length, width, and depth (or height).

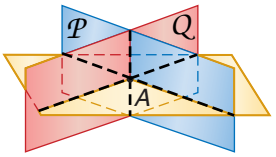
IDENTIFY THREE-DIMENSIONAL FIGURES

A **plane** is a two-dimensional flat surface that extends in all directions. There are different ways that planes may be related in space.

Intersect in a Line



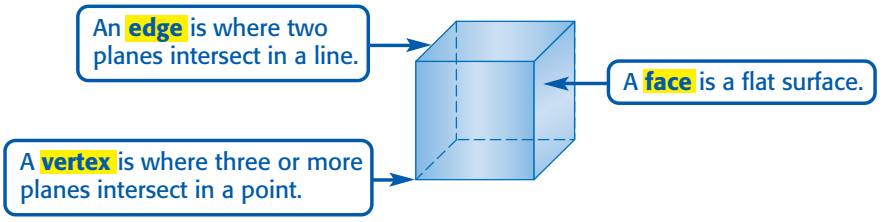
Intersect in a Point



No Intersection

These are called *parallel planes.*

Intersecting planes can also form three-dimensional figures or **solids**. A **polyhedron** is a solid with flat surfaces that are polygons.

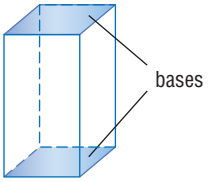


A **prism** is a solid with two congruent parallel polygonal **bases**. The remaining edges join corresponding vertices of the bases so that the remaining faces are rectangles.

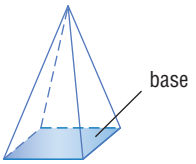
The famous pyramids of Egypt have a shape representative of solids called pyramids. A **pyramid** is a solid with a polygonal base lying in one plane plus one additional vertex not lying on that plane. The remaining edges of the pyramid join the additional vertex to the vertices of the base.

Prisms and pyramids are named by the shape of their bases.

prism



pyramid

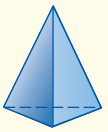
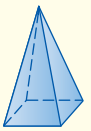


Study Tip

Common Misconception

In a rectangular prism, the bases do not have to be on the top and bottom. Any two parallel rectangles are bases. In a triangular pyramid, any face is a base.

Key Concept — *Polyhedrons*

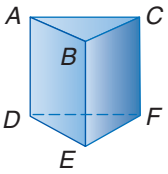
Polyhedron	triangular prism	rectangular prism	triangular pyramid	rectangular pyramid
Number of Bases	2	2	1	1
Polygon Base	triangle	rectangle	triangle	rectangle
Figure				

Use the labels on the vertices to name a base or a face of a solid.

Example 1 Identify Prisms and Pyramids

Identify each solid. Name the bases, faces, edges, and vertices.

a.



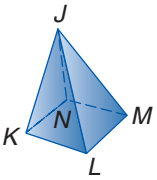
This figure has two parallel congruent bases that are triangles, *ABC* and *DEF*, so it is a triangular prism.

faces: *ABC*, *ADEB*, *BEFC*, *CFDA*, *DEF*

edges: $\overline{AB}$, $\overline{BC}$, $\overline{CA}$, $\overline{AD}$, $\overline{BE}$, $\overline{CF}$, $\overline{DE}$, $\overline{EF}$, $\overline{FD}$

vertices: *A*, *B*, *C*, *D*, *E*, *F*

b.



This figure has one rectangular base, *KLMN*, so it is a rectangular pyramid.

faces: *JKL*, *JLM*, *JMN*, *JNK*, *KLMN*

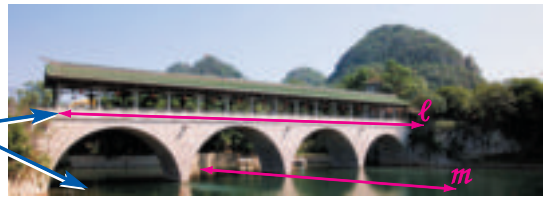
edges: $\overline{JK}$, $\overline{JL}$, $\overline{JM}$, $\overline{JN}$, $\overline{NK}$, $\overline{KL}$, $\overline{LM}$, $\overline{MN}$

vertices: *J*, *K*, *L*, *M*, *N*

Concept Check How many faces does a cube have?

DIAGONALS AND SKEW LINES **Skew lines** are lines that are neither intersecting nor parallel. They lie in different planes. Line ℓ along the bridge below and line m along the river beneath it are skew.

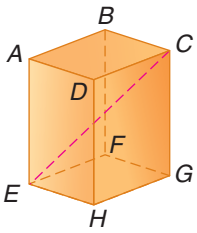
The river is not parallel to the bridge and it does not touch the top of the bridge.



Study Tip

Diagonals
Three other diagonals could have been drawn in the prism at the right: $\overline{AG}$, $\overline{BH}$, and $\overline{DF}$.

We can use rectangular prisms to show skew lines. $\overline{EC}$ is a *diagonal* of the prism at the right because it joins two vertices that have no faces in common. $\overline{EC}$ is skew to $\overline{AD}$.

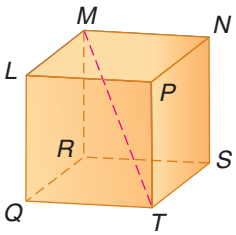


Example 2 Identify Diagonals and Skew Lines

Identify a diagonal and name all segments that are skew to it.

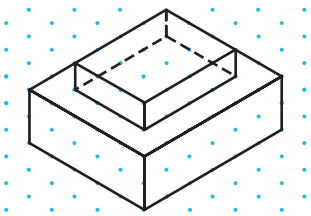
$\overline{MT}$ is a diagonal because vertex M and vertex T do not intersect any of the same faces.

$\overline{LP}$, $\overline{PN}$, $\overline{LQ}$, $\overline{NS}$, $\overline{QR}$, and $\overline{RS}$ are skew to $\overline{MT}$.



Example 3 Analyze Real-World Drawings

ARCHITECTURE **An architect's sketch shows the plans for a new office building. Each unit on the drawing represents 40 feet.**



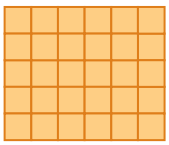
a. Draw a top view and find the area of the ground floor.

The drawing is 6×5, so the actual dimensions are $6(40) \times 5(40)$ or 240 feet by 200 feet.

$A = \ell \cdot w$ Formula for area

$A = 240 \cdot 200$ or 48,000

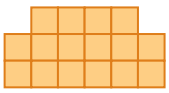
The area of the ground floor is 48,000 square feet.



top view

b. How many floors are in the office building if each floor is 15 feet high?

You can see from the side view that the height of the building is 3 units.



side view

total height: 3 units $\times$ 40 feet per unit $=$ 120 feet

number of floors: 120 feet $\div$ 15 feet per floor $=$ 8 floors

There are 8 floors in the office building.

Check for Understanding

Concept Check

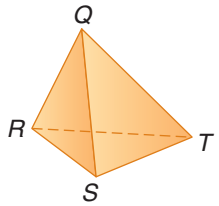
1. **Describe** the number of planes that form a square pyramid and discuss how the planes form edges and vertices of the pyramid.

2. **OPEN ENDED** Choose a solid object from your home and give an example and a nonexample of edges that form skew lines. Include drawings of your example and nonexample.

Guided Practice

Identify each solid. Name the bases, faces, edges, and vertices.

3.



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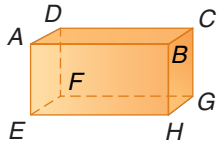
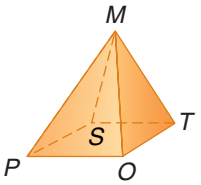


For Exercises 5 and 6, use the rectangular pyramid shown at the right.

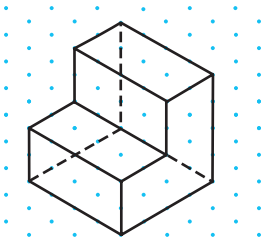


5. State whether $\overline{MP}$ and $\overline{TM}$ are *parallel*, *skew*, or *intersecting*.

6. Identify all lines skew to $\overline{PO}$.

Application

BUILDING **The sketch at the right shows the plans for porch steps.**



7. Draw and label the top, front, and side views.

8. If each unit on the drawing represents 4 inches, what is the height of the steps in feet?

Practice and Apply

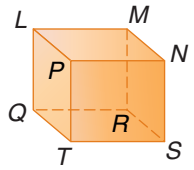
Homework Help

For Exercises	See Examples
9–12, 19–21	1
13–16, 22–25	2
17, 18	3

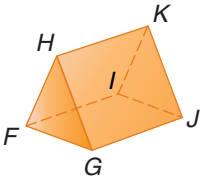
Extra Practice
See page 750.

Identify each solid. Name the bases, faces, edges, and vertices.

9.



10.



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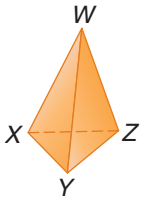

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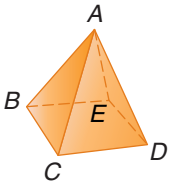
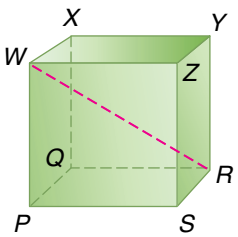


For Exercises 13–16, use the rectangular prism.



13. Identify a diagonal.
14. Name four segments skew to $\overline{QR}$.
15. State whether $\overline{WR}$ and $\overline{XY}$ are *parallel*, *skew*, or *intersecting*.
16. Name a segment that does *not* intersect the plane that contains *WXYZ*.

COMICS **For Exercises 17 and 18, use the comic below.**

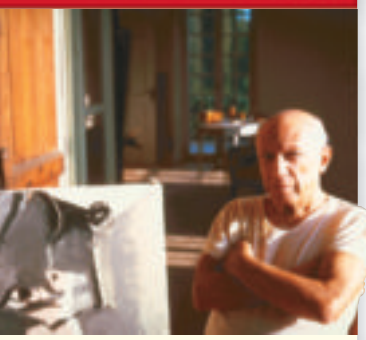
SHOE

17. Which view of the Washington Monument is shown?
18. **RESEARCH** Use the Internet or another source to find a photograph of the Washington Monument. Draw and label the top, side, and front views.

ART **For Exercises 19 and 20, refer to Picasso's painting *The Factory, Horta de Ebro* shown at the right.**

19. Describe the polyhedrons shown in the painting.
20. Explain how the artist portrayed three-dimensional objects on a flat canvas.
21. **RESEARCH** Find other examples of art in which polyhedrons are shown. Describe the polyhedrons.

More About . . .



Art

Pablo Picasso (1881–1973) was one of the developers of a movement in art called *Cubism*. Cubist paintings are characterized by their angular shapes and sharp edges.

Source: World Wide Arts Resources

Determine whether each statement is *sometimes*, *always*, or *never* true. Explain.

22. Any two planes intersect in a line.
23. Two planes intersect in a single point.
24. A pyramid contains a diagonal.
25. Three planes do not intersect in a point.
26. **CRITICAL THINKING** Use isometric dot paper to draw a three-dimensional figure in which the front and top views have a line of symmetry but the side view does not. Then discuss whether the figure has bilateral symmetry or rotational symmetry.

27. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How are 2-dimensional figures related to 3-dimensional figures?

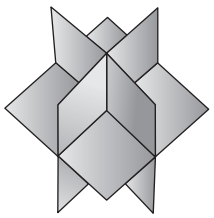
Include the following in your answer:

- an explanation of the difference between 2-dimensional figures and 3-dimensional figures, and
- a description of how 2-dimensional figures can form a 3-dimensional figure.

Standards Practice

Standardized Test Practice

28. Determine the intersection of the three planes at the right.



(A) point

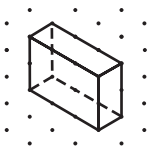
(B) line

(C) plane

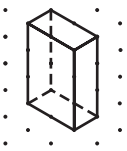
(D) no intersection

29. Which figure does *not* have the same dimensions as the other figures?

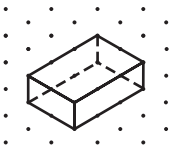
(A)



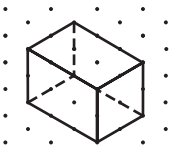
(B)



(C)



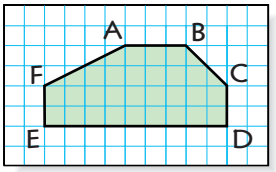
(D)



Maintain Your Skills

Mixed Review

30. Find the area of *ABCDEF* if each unit represents 1 square centimeter. *(Lesson 10-10)*



31. Find the circumference and the area of a circle whose radius is 6 centimeters. Round to the nearest tenth. *(Lesson 10-9)*

Use a calculator to find each ratio to the nearest ten thousandth. *(Lesson 9-7)*

32. sin 35° **33.** sin 30° **34.** cos 280°

Solve each inequality. Check your solution. *(Lesson 7-4)*

35. $c + 4 < 12$ **36.** $7 \geq t - 2$ **37.** $-26 < n + (-15)$

38. $k + (-4) \geq 3.8$ **39.** $y - \frac{1}{4} < 1\frac{1}{2}$ **40.** $3\frac{1}{5} > a - \frac{3}{10}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find the area of each triangle described.**

*(To review **areas of triangles**, see Lesson 10-5.)*

41. base, 4 in.; height, 7 in. **42.** base, 10 ft; height, 9 ft

43. base, 6.5 cm; height, 2 cm **44.** base, 0.4 m; height, 1.3 m

Geometry Activity

A Preview of Lesson 11-2

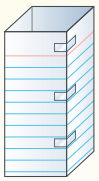
Volume

In this activity, you will investigate volume by making containers of different shapes and comparing how much each container holds.

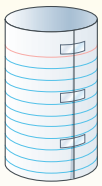
Activity

Collect the Data

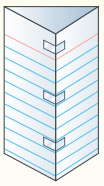
Step 1 Use three 5-inch × 8-inch index cards to make three different containers, as shown below.



square base with 2-inch sides

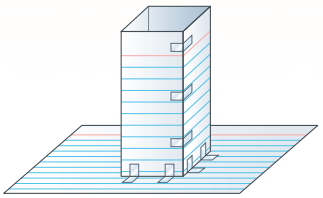


circular base with 8-inch circumference



triangular base with sides 2 inches, 3 inches, and 3 inches

Step 2 Tape one end of each container to another card as a bottom, but leave the top open, as shown at the right.



Step 3 Estimate which container would hold the most (have the greatest volume) and which would hold the least (have the least volume), or whether all the containers would hold the same amount.

Step 4 Use rice to fill the container that you believe holds the least amount. Then pour the rice from this container into another container. Does the rice fill the second container? Continue the process until you find out which container, if any, has the least volume and which has the greatest.

Analyze the Data

1. Which container holds the greatest amount of rice? Which holds the least amount?
2. How do the heights of the three containers compare? What is each height?
3. Compare the perimeters of the bases of each container. What is each base perimeter?
4. Trace the base of each container onto grid paper. Estimate the area of each base.
5. Which container has the greatest base area?
6. Does there appear to be a relationship between the area of the bases and the volume of the containers when the heights remain unchanged? Explain.

11-2 Volume: Prisms and Cylinders

What You'll Learn

- Find volumes of prisms.
- Find volumes of circular cylinders.

Vocabulary

- volume
- cylinder

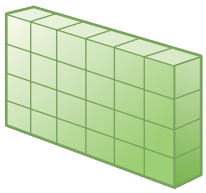
California Standards

Standard 7MG1.1 Compare weights, **capacities, geometric measures,** times, and temperatures **within** and between **measurement systems** (e.g., miles per hour and feet per second, cubic inches to cubic centimeters). (CAHSEE)

Standard 7MG2.4 Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and **to conversions between units (1 square foot = 144 square inches or [1 ft^2] = [144 in^2],** 1 cubic inch is approximately 16.38 cubic centimeters or [1 in^3] = [16.38 cm^3]). (CAHSEE)

How is volume related to area?

The rectangular prism is built from 24 cubes.



a. Build three more rectangular prisms using 24 cubes. Enter the dimensions and base areas in a table.

Prism	Length (units)	Width (units)	Height (units)	Area of Base (units^2)
1	6	1	4	6
2				
3				
4				

b. *Volume* equals the number of cubes that fill a prism. How is the volume of each prism related to the product of the length, width, and height?

c. **Make a conjecture** about how the area of the base B and the height h are related to the volume V of a prism.

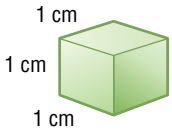
VOLUMES OF PRISMS The prism above has a volume of 24 cubic centimeters. **Volume** is the measure of space occupied by a solid region.

To find the volume of a prism, you can use the area of the base and the height, as given by the following formula.

Study Tip

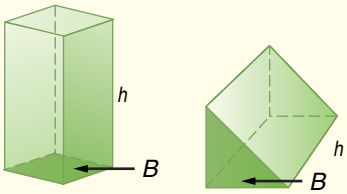
Measures of Volume

A cubic centimeter (cm^3) is a cube whose edges measure 1 centimeter.



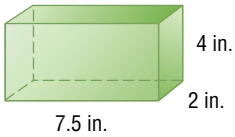
Key Concept — Volume of a Prism

- **Words** The volume V of a prism is the area of the base B times the height h.
- **Symbols** $V = Bh$
- **Models**



Example 1 Volume of a Rectangular Prism

Find the volume of the prism.

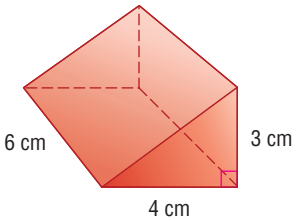


$V = Bh$	Formula for volume of a prism
$V = (\ell \cdot w)h$	The base is a rectangle, so $B = \ell \cdot w$.
$V = (7.5 \cdot 2)4$	$\ell = 7.5$, $w = 2$, $h = 4$
$V = 60$	Simplify.

The volume is 60 cubic inches.

Example 2 Volume of a Triangular Prism

Find the volume of the triangular prism.



$V = Bh$	Formula for volume of a prism
$V = \left(\frac{1}{2} \cdot 4 \cdot 3\right)h$	B = area of base or $\frac{1}{2} \cdot 4 \cdot 3$
$V = \left(\frac{1}{2} \cdot 4 \cdot 3\right)6$	The height of the prism is 6 cm.
$V = 36$	Simplify.

The volume is 36 cubic centimeters.

Example 3 Height of a Prism

AQUARIUMS **A wall is being constructed to enclose three sides of an aquarium that is a rectangular prism 8 feet long and 5 feet wide. If the aquarium is to contain 220 cubic feet of water, what is its height?**

$V = Bh$	Formula for volume of a prism
$V = \ell \cdot w \cdot h$	Formula for volume of a rectangular prism
$220 = 8 \cdot 5 \cdot h$	Replace V with 220, ℓ with 8, and w with 5.
$220 = 40h$	Simplify.
$5.5 = h$	Divide each side by 40.

The height of the aquarium is 5.5 feet.

To find the volume of a solid with several prisms, break it into parts.

Standards Practice

Standardized Test Practice

Example 4 Volume of a Complex Solid

Multiple-Choice Test Item

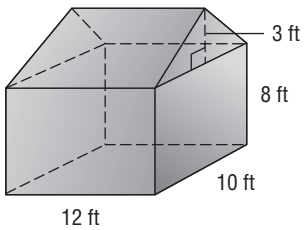
Find the volume of the solid at the right.

(A) 180 ft^3

(B) 1320 ft^3

(C) 960 ft^3

(D) 1140 ft^3



Read the Test Item

The solid is made up of a rectangular prism and a triangular prism. The *volume of the solid* is the sum of both volumes.

Solve the Test Item

Step 1 The volume of the rectangular prism is 12(10)(8) or 960 ft^3.

Step 2 In the triangular prism, the area of the base is $\frac{1}{2}(10)(3)$, and the height is 12. Therefore, the volume is $\frac{1}{2}(10)(3)(12)$ or 180 ft^3.

Step 3 Add the volumes.

$960\ ft^3 + 180\ ft^3 = 1140\ ft^3$

The answer is D.

Test-Taking Tip

Estimate You can eliminate A and C as answers because the volume of the rectangular prism is 960 ft^3, so the volume of the whole solid must be greater.

Reading Math

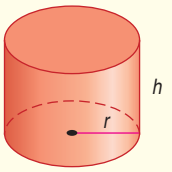
Cylinders

In this text, *cylinder* refers to a cylinder with a circular base.

VOLUMES OF CYLINDERS Cans of soup and oil drums are examples of cylindrical solids. A **cylinder** is a solid with two parallel congruent circular bases. The third surface of the cylinder consists of all parallel circles of the same radius whose centers lie on the segment joining the centers of the bases. In this way cylinders resemble rectangular prisms, as they have identical cross sections.

Key Concept — Volume of a Cylinder

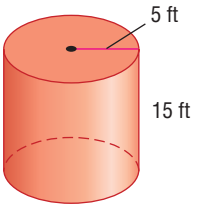
- **Words** The volume V of a cylinder with radius r is the area of the base B times the height h.
- **Symbols** $V = Bh$ or $V = \pi r^2 h$, where $B = \pi r^2$
- **Model**



Example 5 Volume of a Cylinder

Find the volume of each cylinder. Round to the nearest tenth.

a.



$V = \pi r^2 h$ Formula for volume of a cylinder

$V = \pi \cdot 5^2 \cdot 15$ Replace r with 5 and h with 15.

$V \approx 1178.1$ Simplify.

The volume is about 1178.1 cubic feet.

b. diameter of base 16.4 mm, height 20 mm

Since the diameter is 16.4 mm, the radius is 8.2 mm.

$V = \pi r^2 h$ Formula for volume of a cylinder

$V = \pi \cdot 8.2^2 \cdot 20$ Replace r with 8.2 and h with 20.

$V \approx 4224.8$ Simplify.

The volume is about 4224.8 cubic millimeters.

Check for Understanding

Concept Check

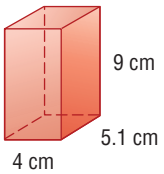
1. **OPEN ENDED** Describe a problem from an everyday situation in which you need to find the volume of a cylinder or a rectangular prism.

2. **FIND THE ERROR** Eric says that doubling the length of each side of a cube doubles the volume. Marissa says the volume is eight times greater. Who is correct? Explain your reasoning.

Guided Practice

Find the volume of each solid. If necessary, round to the nearest tenth.

3.



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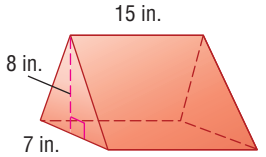

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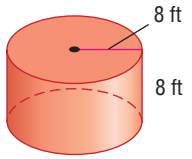


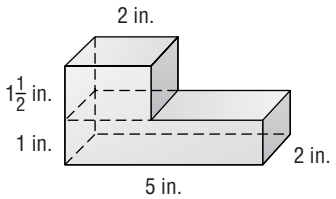
6. rectangular prism: length 6 in., width 6 in., height 9 in.
7. cylinder: radius 3 yd, height 10 yd
8. Find the height of a rectangular prism with a length of 3 meters, width of 1.5 meters, and a volume of 60.3 cubic meters.

9. **ENGINEERING** A cylindrical storage tank is being manufactured to hold at least 1,000,000 cubic feet of natural gas and have a diameter of no more than 80 feet. What height should the tank be to the nearest tenth of a foot?

Standards Practice
Standardized Test Practice

10. Find the volume of the solid at the right.

(A) 6 in^3 (B) 10 in^3
(C) 13 in^3 (D) 16 in^3



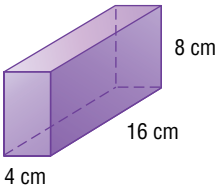
Practice and Apply

Homework Help

For Exercises	See Examples
11, 12, 17, 23–27	1
13, 14, 18	2
15, 16, 19, 28	5
21, 22	3

Extra Practice
See page 750.

Find the volume of each solid shown or described. If necessary, round to the nearest tenth.

11.


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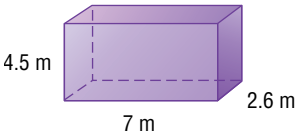

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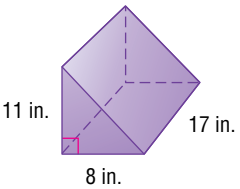

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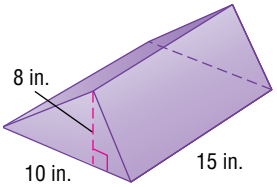

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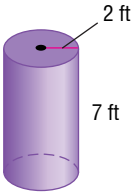

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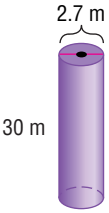


17. rectangular prism: length 3 mm, width 5 mm, height 15 mm

18. triangular prism: base of triangle 8 in., altitude of triangle 15 in., height of prism $6\frac{1}{2}$ in.

19. cylinder: $d = 2.6$ m, $h = 3.5$ m

20. octagonal prism: base area 25 m^2, height 1.5 m

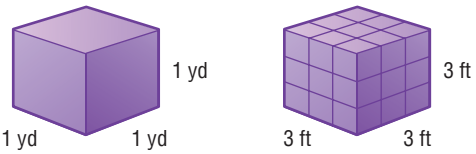
21. Find the height of a rectangular prism with a length of 4.2 meters, width of 3.2 meters, and volume of 83.3 m^3.

22. Find the height of a cylinder with a radius of 2 feet and a volume of 28.3 ft^3.

CONVERTING UNITS OF MEASURE

For Exercises 23–25, use the cubes at the right.

The volume of the left cube is 1 yd^3. In the right cube, only the units have been changed. So, 1 yd^3 = 3(3)(3) or 27 ft^3. Use a similar process to convert each measurement.



23. 1 ft^3 = ■ in^3

24. 1 cm^3 = ■ mm^3

25. 1 m^3 = ■ cm^3

26. **METALS** The *density* of gold is 19.29 grams per cubic centimeter. Find the mass in grams of a gold bar that is 2 centimeters by 3 centimeters by 2 centimeters.

27. **MICROWAVES** The inside of a microwave oven has a volume of 1.2 cubic feet and measures 18 inches wide and 10 inches long. To the nearest tenth, how deep is the inside of the microwave? (*Hint*: Convert 1.2 cubic feet to cubic inches.)

28. **BATTERIES** The current of an alkaline battery corresponds to its volume. Find the volume of each cylinder-shaped battery shown in the table. Write each volume in cm^3. (*Hint*: 1 cm^3 = 1000 mm^3)

Battery Size	Diameter (mm)	Height (mm)
D	33.3	61.1
C	25.5	50.0
AA	14.5	50.5
AAA	10.5	44.5

29. **CRITICAL THINKING** An $8\frac{1}{2}$-by-11-inch piece of paper is rolled to form a cylinder. Will the volume be greater if the height is $8\frac{1}{2}$ inches or 11 inches, or will the volumes be the same? Explain your reasoning.

30. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is volume related to area?

Include the following in your answer:

- an explanation of why area is given in square units and volume is given in cubic units, and
- a description of why the formula for volume includes area.

Standards Practice

Standardized Test Practice

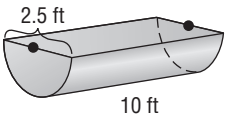
31. Which is the best estimate for the volume of a cube whose sides measure 18.79 millimeters?

(A) 80 mm^3 (B) 800 mm^3 (C) 8000 mm^3 (D) 80,000 mm^3

32. Find the volume of the figure at the right.

(A) 24.5 ft^3 (B) 20.5 ft^3

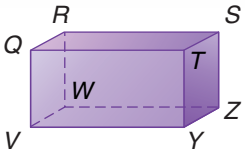
(C) 48 ft^3 (D) 49 ft^3



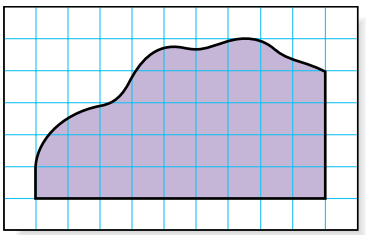
Maintain Your Skills

Mixed Review

33. Identify a pair of skew lines in the prism at the right. *(Lesson 11-1)*



34. Estimate the area of the shaded figure to the nearest square unit. *(Lesson 10-8)*



Solve each inequality. Check your solution. *(Lesson 7-4)*

35. $x + 5 > -3$ 36. $k + (-9) \geq 1.8$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each product.**

*(To review **multiplying fractions**, see Lesson 5-3.)*

37. $\frac{1}{3} \cdot 5 \cdot 15$ 38. $\frac{1}{3} \cdot 4 \cdot 9$ 39. $\frac{1}{3} \cdot 2 \cdot 2 \cdot 3$

40. $\frac{1}{3} \cdot 3 \cdot 4 \cdot 8$ 41. $\frac{1}{3} \cdot 2^2 \cdot 21$ 42. $\frac{1}{3} \cdot 3^2 \cdot 10$

11-3 Volume: Pyramids and Cones

What You'll Learn

- Find volumes of pyramids.
- Find volumes of cones.

Vocabulary

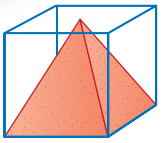
- cone

California Standards

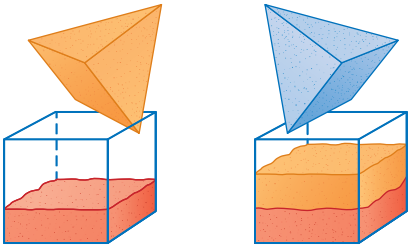
Standard 7MG2.1 Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and **volume of basic three-dimensional figures,** including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders. (CAHSEE)

How is the volume of a pyramid related to the volume of a prism?

You can see that the volume of the pyramid shown at the right is less than the volume of the prism in which it sits.



If the pyramid were made of sand, it would take three pyramids to fill a prism having the same base dimensions and height.



a. Compare the base areas and compare the heights of the prism and the pyramid.

b. How many times greater is the volume of the prism than the volume of one pyramid?

c. What fraction of the prism volume does one pyramid fill?

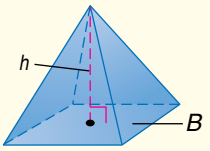
Study Tip

Height of Pyramid
The height of a pyramid is the distance from the vertex, perpendicular to the base.

VOLUMES OF PYRAMIDS A pyramid has one-third the volume of a prism with the same base area and height.

Key Concept — Volume of a Pyramid

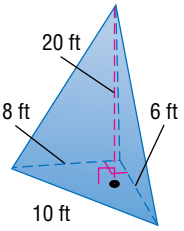
- **Words** The volume V of a pyramid is one-third the area of the base B times the height h.
- **Symbols** $V = \frac{1}{3}Bh$
- **Model**



Example 1 Volumes of Pyramids

Find the volume of each pyramid. If necessary, round to the nearest tenth.

a.



$V = \frac{1}{3}Bh$	Formula for volume of a pyramid
$V = \frac{1}{3}\left(\frac{1}{2} \cdot 8 \cdot 6\right)h$	The base is a triangle, so $B = \frac{1}{2} \cdot 8 \cdot 6$.
$V = \frac{1}{3}\left(\frac{1}{2} \cdot 8 \cdot 6\right)20$	The height of the pyramid is 20 feet.
$V = 160$	Simplify.

The volume is 160 cubic feet.

b. base area 125 cm², height 6.5 cm

$V = \frac{1}{3}Bh$ Formula for volume of a pyramid

$V = \frac{1}{3}(125)(6.5)$ Replace B with 125 and h with 6.5.

$V \approx 270.8$ Simplify.

The volume is about 270.8 cubic centimeters.

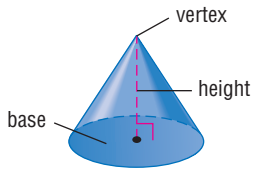
Reading Math

Cones

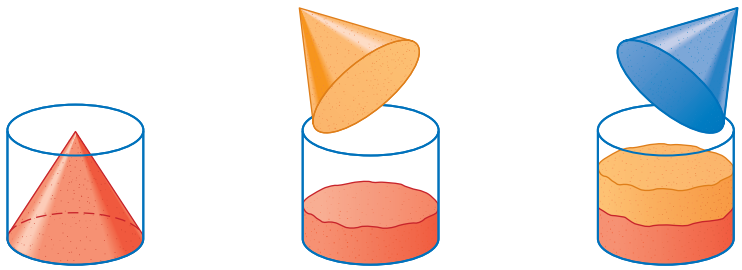
In this text, *cone* refers to a circular cone.

VOLUMES OF CONES

A **cone** is a solid with a circular base lying in one plane plus a vertex not lying on that plane. The remaining surface of the cone is formed by joining the vertex to points on the circle by line segments.



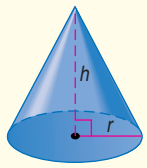
The volumes of a cone and a cylinder are related in the same way as the volumes of a pyramid and a prism are related.



The volume of a cone is $\frac{1}{3}$ the volume of a cylinder with the same base area and height.

Key Concept — Volume of a Cone

- **Words** The volume V of a cone with radius r is one-third the area of the base B times the height h.
- **Symbols** $V = \frac{1}{3}Bh$ or $V = \frac{1}{3}\pi r^2 h$, where $B = \pi r^2$
- **Model**



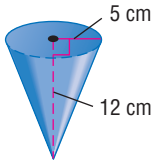
Example 2 Volume of a Cone

Find the volume of the cone. Round to the nearest tenth.

$V = \frac{1}{3}\pi r^2 h$ Formula for volume of a cone

$V = \frac{1}{3} \cdot \pi \cdot 5^2 \cdot 12$ Replace r with 5 and h with 12.

$V \approx 314.2$ Simplify.

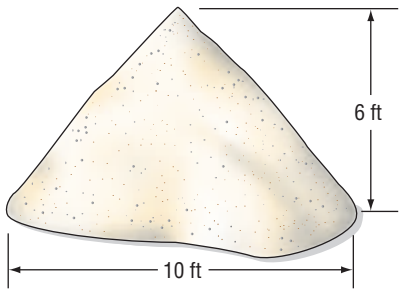


The volume is about 314.2 cubic centimeters.

Concept Check What is the volume of a cone whose base area is 86 meters squared and whose height is 3 meters?

Example 3 Use Volume to Solve Problems

HIGHWAY MAINTENANCE **Salt and sand mixtures are often used on icy roads. When the mixture is dumped from a truck into the staging area, it forms a cone-shaped mound with a diameter of 10 feet and a height of 6 feet.**



a. What is the volume of the salt-sand mixture?

Estimate: $\frac{1}{3} \cdot 3 \cdot 5^2 \cdot 6 = 150$

$V = \frac{1}{3}\pi r^2 h$ Formula for volume of a cone

$V = \frac{1}{3} \cdot \pi \cdot 5^2 \cdot 6$ Since $d = 10$, replace r with 5. Replace h with 6.

$V \approx 157$

The volume of the mixture is about 157 cubic feet.

b. How many square feet of roadway can be salted using the mixture in part a if 500 square feet can be covered by 1 cubic foot of salt?

$$\text{ft}^2 \text{ of roadway} = 157 \text{ ft}^3 \text{ mixture} \times \frac{500 \text{ ft}^2 \text{ of roadway}}{1 \text{ ft}^3 \text{ mixture}}$$

$$= 78{,}500 \text{ ft}^2 \text{ of roadway}$$

So, 78,500 square feet of roadway can be salted.

Study Tip

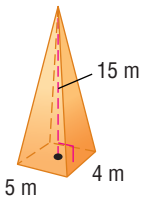
Look Back
To review **dimensional analysis**, see Lesson 5-3.

Check for Understanding

Concept Check

1. **Explain** why you can use πr^2 to find the area of the base of a cone.
2. **List** the formulas for volume that you have learned so far in this chapter. Tell what the variables represent and explain how you can remember which formula goes with which solid.
3. **OPEN ENDED** Draw and label a cone whose volume is between 100 cm^3 and 1000 cm^3.

Guided Practice **Find the volume of each solid. If necessary, round to the nearest tenth.**

4.


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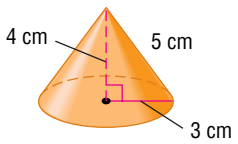

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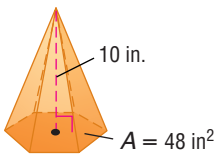


7. rectangular pyramid: length 9 ft, width 7 ft, height 18 ft
8. cone: radius 4 mm, height 6.5 mm

Application

9. **HISTORY** The Great Pyramid of Khufu in Egypt was originally 481 feet high and had a square base 756 feet on a side. What was its volume? Use an estimate to check your answer.

Practice and Apply

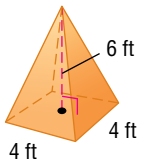
Homework Help

For Exercises	See Examples
10–22	1, 2
23–25	3

Extra Practice
See page 751.

Find the volume of each solid. If necessary, round to the nearest tenth.

10.



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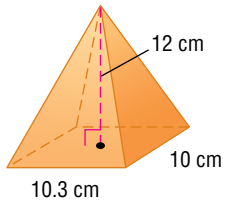

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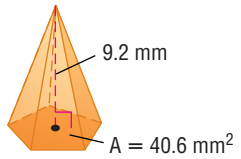

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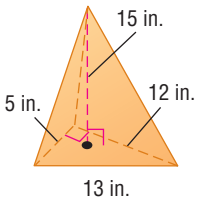
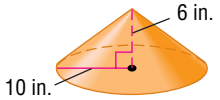
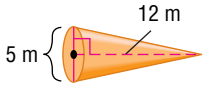


14.



15.



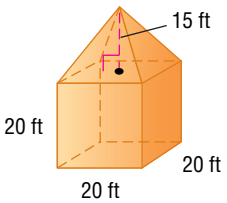
16. square pyramid: length 5 in., height 6 in.

17. hexagonal pyramid: base area 125 cm^2, height 6.5 cm

18. cone: radius 3 yd, height 14 yd

19. cone: diameter 12 m, height 15 m

20.



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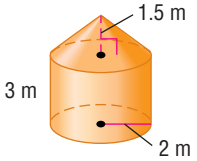
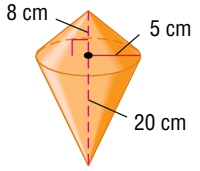


22.



23. **GEOLOGY** A stalactite in the Endless Caverns in Virginia is cone-shaped. It is 4 feet long and has a diameter at its base of 1.5 feet.

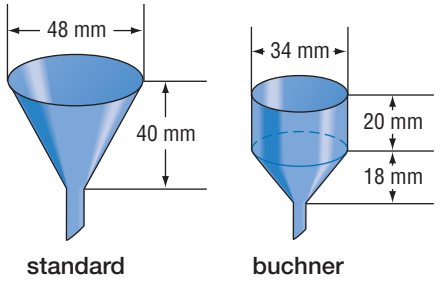
a. Find the volume of the stalactite to the nearest tenth.

b. The stalactite is made of calcium carbonate, which weighs 131 pounds per cubic foot. What is the weight of the stalactite?

WebQuest

Finding the volumes of three-dimensional figures will help you analyze structures. Visit www.pre-alg.com/webquest to continue work on your WebQuest project.

24. **SCIENCE** In science, a standard funnel is shaped like a cone, and a buchner funnel is shaped like a cone with a cylinder attached to the base. Which funnel has the greatest volume?



25. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is the volume of a pyramid related to the volume of a prism?

Include the following in your answer:

- a discussion of the similarities between the dimensions and base area of the pyramid and prism shown at the beginning of the lesson, and
- a description of how the formulas for the volume of a pyramid and the volume of a prism are similar.

26. CRITICAL THINKING

a. If you double the height of a cone, how does the volume change?

b. If you double the radius of the base of a cone, how does the volume change? Explain.

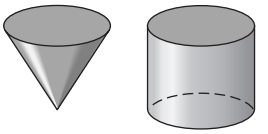
Standards Practice

Standardized Test Practice

27. Choose the best estimate for the volume of a rectangular pyramid 4.9 centimeters long, 3 centimeters wide, and 7 centimeters high.

Ⓐ 7 cm^3 Ⓑ 35 cm^3 Ⓒ 70 cm^3 Ⓓ 105 cm^3

28. The solids at the right have the same base area and height. If the cone is filled with water and poured into the cylinder, how much of the cylinder would be filled?



Ⓐ $\frac{3}{4}$ Ⓑ $\frac{1}{2}$ Ⓒ $\frac{2}{3}$ Ⓓ $\frac{1}{3}$

Extending the Lesson

The volume V of a sphere with radius r is given by the formula $V = \frac{4}{3}\pi r^3$. Find the volume of each sphere to the nearest tenth.

29. radius 6 cm **30.** radius 1.4 in. **31.** diameter 5.9 mm

Maintain Your Skills

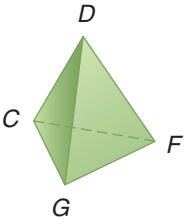
Mixed Review

Find the volume of each prism or cylinder. If necessary, round to the nearest tenth. *(Lesson 11-2)*

32. rectangular prism: length 4 cm, width 8 cm, height 2 cm

33. cylinder: diameter 1.6 in., height 5 in.

34. Identify the solid at the right. Name the bases, faces, edges, and vertices. *(Lesson 11-1)*



35. Find the distance between $A(3, 7)$ and $B(-2, 1)$. Round to the nearest tenth, if necessary. *(Lesson 9-6)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Estimate each product.**

*(To review **estimating products**, see page 714.)*

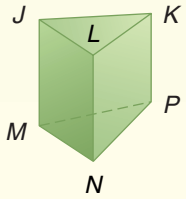
36. $4.9 \cdot 5.1 \cdot 3$ **37.** $2 \cdot 1.7 \cdot 9$ **38.** $2 \cdot \pi \cdot 6.8$

Practice Quiz 1

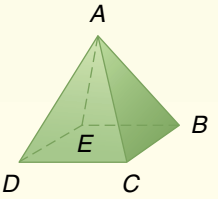
Lessons 11-1 through 11-3

Identify each solid. *(Lesson 11-1)*

1.



2.



Find the volume of each solid. If necessary, round to the nearest tenth. *(Lesson 11-2)*

3. cylinder: radius 2 cm, height 1 cm **4.** hexagonal prism: base area 42 ft^2, height 18 ft

5. MINING An open pit mine in the Elk mountain range is cone-shaped. The mine is 420 feet across and 250 feet deep. What volume of material was removed? *(Lesson 11-3)*

11-4 Surface Area: Prisms and Cylinders

What You'll Learn

- Find surface areas of prisms.
- Find surface areas of cylinders.

Vocabulary

- surface area

California Standards
Standard 7MG2.1 Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the **surface area** and volume **of basic three-dimensional figures, including** rectangles, parallelograms, trapezoids, squares, triangles, circles, **prisms, and cylinders.** (CAHSEE)

How is the surface area of a solid different from its volume?

The sizes and prices of shipping boxes are shown in the table.

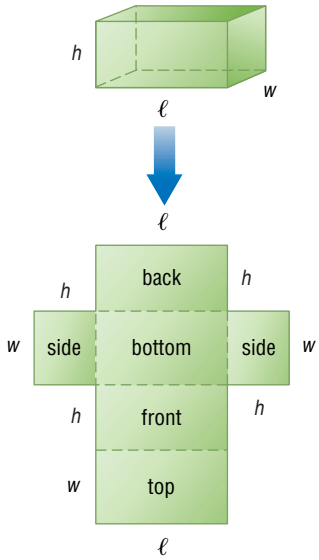
Box	Size (in.)	Price ($)
A	$8 \times 8 \times 8$	$1.50
B	$15 \times 10 \times 12$	$2.25
C	$20 \times 14 \times 10$	$3.00

a. For each box, find the area of each face and the sum of the areas.

b. Find the volume of each box. Are these values the same as the values you found in part **a**? Explain.

SURFACE AREAS OF PRISMS

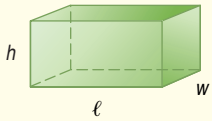
If you open up a box or prism to form a net, you can see all the surfaces. The sum of the areas of these surfaces is called the **surface area** of the prism.



Faces	Area
top and bottom	$(\ell \cdot w) + (\ell \cdot w) = 2\ell w$
front and back	$(\ell \cdot h) + (\ell \cdot h) = 2\ell h$
two sides	$(w \cdot h) + (w \cdot h) = 2wh$
Sum of areas $\rightarrow$	$2\ell w + 2\ell h + 2wh$ or $2(\ell w + \ell h + wh)$

Key Concept — Surface Area of Rectangular Prisms

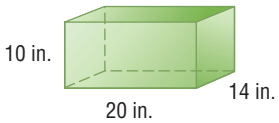
- **Words** The surface area S of a rectangular prism with length ℓ, width w, and height h is the sum of the areas of the faces.
- **Model**



- **Symbols** $S = 2\ell w + 2\ell h + 2wh = 2(\ell w + \ell h + wh)$

Example 1 Surface Area of a Rectangular Prism

Find the surface area of the rectangular prism.

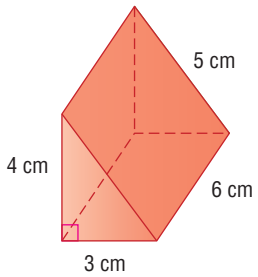


$S = 2\ell w + 2\ell h + 2wh$	Write the formula.
$S = 2(20)(14) + 2(20)(10) + 2(14)(10)$	Substitution
$S = 1240$	Simplify.

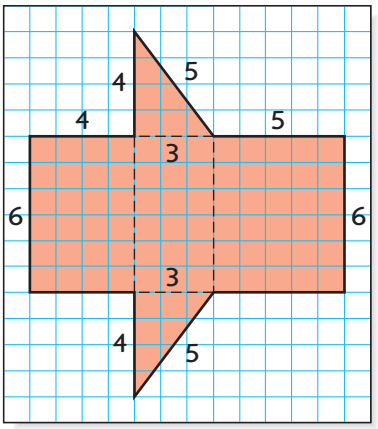
The surface area of the prism is 1240 square inches.

Example 2 Surface Area of a Triangular Prism

Find the surface area of the triangular prism.



One way to easily see all of the surfaces of the prism is to draw a net on grid paper and label the dimensions of each face.



Find the area of each face.

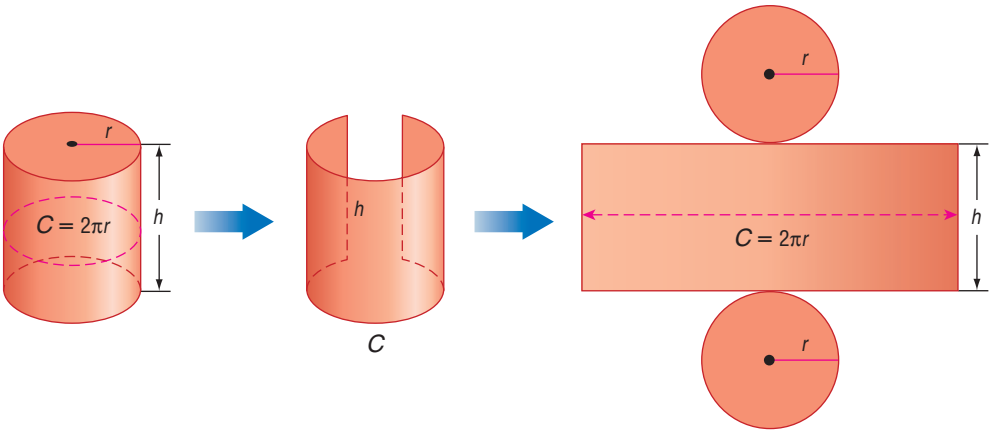
bottom $3 \cdot 6 = 18$
left side $4 \cdot 6 = 24$
right side $5 \cdot 6 = 30$
$\Big\}$ $A = \ell w$

two bases $2\left(\frac{1}{2} \cdot 3 \cdot 4\right) = 12$ $\quad A = \frac{1}{2}bh$

Add to find the total surface area.
$18 + 24 + 30 + 12 = 84$

The surface area of the triangular prism is 84 square centimeters.

SURFACE AREAS OF CYLINDERS You can also find surface areas of cylinders. If you unroll a cylinder, its net is a rectangle and two circles.



Model		Net
bases	→	congruent circles
curved surface	→	rectangle
height h	→	width of rectangle
circumference C	→	length of rectangle

The area of each circular base is πr^2. The area of the rectangular region is $\ell \cdot w$, or $2\pi r \cdot h$.

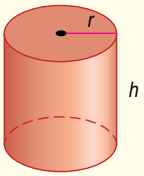
$$\underbrace{S}_{\text{The surface area of a cylinder}} \underbrace{=}_{\text{equals}} \underbrace{2(\pi r^2)}_{\text{the area of two circular bases}} \underbrace{+}_{\text{plus}} \underbrace{2\pi rh}_{\text{the area of the curved surface.}}$$

Study Tip

Circles and Rectangles
To see why $\ell = 2\pi r$, find the circumference of a soup can by using the formula $C = 2\pi r$. Then peel off its label and measure the length (minus the overlapping parts).

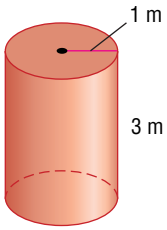
Key Concept — Surface Area of Cylinders

- **Words** The surface area S of a cylinder with height h and radius r is the area of the two bases plus the area of the curved surface.
- **Symbols** $S = 2\pi r^2 + 2\pi rh$
- **Model**



Example 3 Surface Area of a Cylinder

Find the surface area of the cylinder. Round to the nearest tenth.



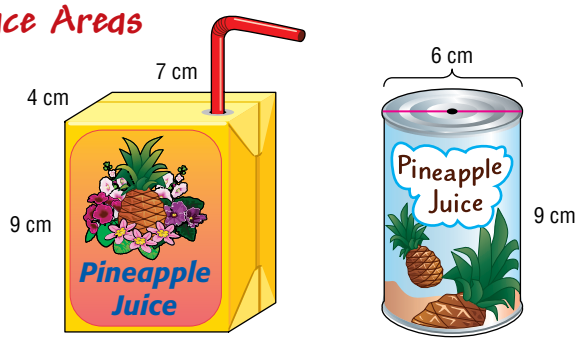
$S = 2\pi r^2 + 2\pi rh$	Formula for surface area of a cylinder
$S = 2\pi(1)^2 + 2\pi(1)(3)$	Replace r with 1 and h with 3.
$S \approx 25.1$	Simplify.

The surface area is about 25.1 square meters.

You can compare surface areas of prisms and cylinders.

Example 4 Compare Surface Areas

FRUIT DRINKS **Both containers hold about the same amount of pineapple juice. Does the box or the can have a greater surface area?**



Surface area of box

$$S = \underbrace{2\ell w}_{\text{top/bottom}} + \underbrace{2\ell h}_{\text{sides}} + \underbrace{2wh}_{\text{front/back}}$$
$$= 2(4 \cdot 7) + 2(4 \cdot 9) + 2(7 \cdot 9)$$
$$= 254$$

Surface area of can

$$S = \underbrace{2\pi r^2}_{\text{top/bottom}} + \underbrace{2\pi rh}_{\text{curved surface}}$$
$$= 2\pi(3)^2 + 2\pi(3)(9)$$
$$\approx 226$$

Since $254 \text{ cm}^2 > 226 \text{ cm}^2$, the box has a greater surface area.

More About . . .

Fruit Drinks
A typical juice box is made of six layers of packaging. These layers include paper, foil, and special adhesives.
Source: www.dupont.com

Concept Check Why might a company prefer to sell juice in cans?

Check for Understanding

Concept Check

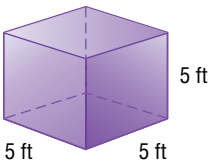
1. **Explain** why surface area is given in square units rather than cubic units.

2. **OPEN ENDED** Find the surface areas of a rectangular prism and a cylinder found in your home.

Guided Practice

Find the surface area of each solid shown or described. If necessary, round to the nearest tenth.

3.



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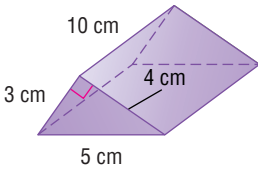

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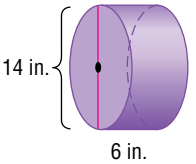


6. rectangular prism: length 3 cm, width 2 cm, height 1 cm

7. cylinder: radius 4 mm, height 1.6 mm

Application

8. **CRAFTS** Brianna sews together pieces of plastic canvas to make tissue box covers. For which tissue box will she use more plastic canvas to cover the sides and the top? Explain.

Box	Length (in.)	Width (in.)	Height (in.)
A	9	4	5
B	5	5	6

Practice and Apply

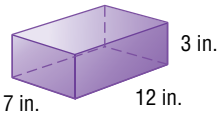
Homework Help

For Exercises	See Examples
9, 10, 15, 16	1
11, 12	2
13, 14, 17–19, 21, 22	3
20	4

Extra Practice
See page 751.

Find the surface area of each solid shown or described. If necessary, round to the nearest tenth.

9.



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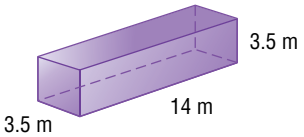

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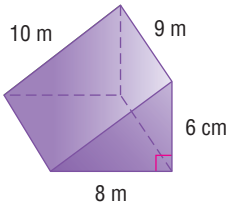

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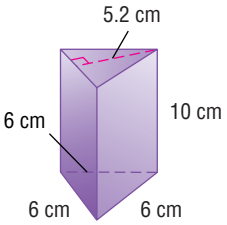

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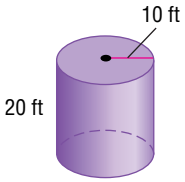

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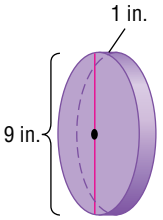


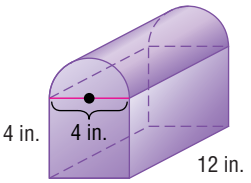
15. cube: side length 7 ft

16. rectangular prism: length 6.2 cm, width 4 cm, height 8.5 cm

17. cylinder: radius 5 in., height 15 in.

18. cylinder: diameter 4 m, height 20 m

19. Find the surface area of the complex solid at the right. Use estimation to check the reasonableness of your answer.



20. **AQUARIUMS** A standard 20-gallon aquarium tank is a rectangular prism that holds approximately 4600 cubic inches of water. The bottom glass needs to be 24 inches by 12 inches to fit on the stand.

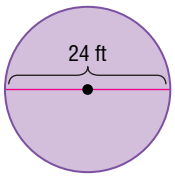
a. Find the height of the aquarium to the nearest inch.

b. Find the total amount of glass needed in square feet for the five faces.

c. An aquarium with an octagonal base has sides that are 9 inches wide and 16 inches high. The area of the base is 392.4 square inches. Do the bottom and sides of this tank have a greater surface area than the rectangular tank? Explain.

POOLS **Vinyl liners cover the inside walls and bottom of the swimming pools whose top views are shown below. Find the area of the vinyl liner for each pool if they are 4 feet deep. Round to the nearest square foot.**

21.



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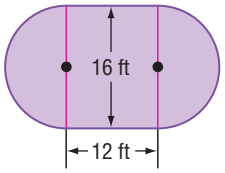


23. **CRITICAL THINKING** Suppose you double the length of the sides of a cube. How is the surface area affected?

24. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is the surface area of a solid different than its volume?

Include the following in your answer:

- a comparison of formulas for surface area and volume, and
- an explanation of the difference between surface area and volume.

Standards Practice

Standardized Test Practice

25. Find the surface area of a cylinder with a diameter of 15 centimeters and height of 2 centimeters.

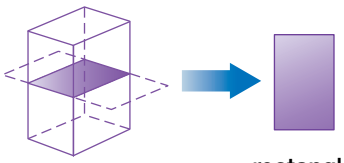
Ⓐ 30 cm^2 Ⓑ 117.8 cm^2 Ⓒ 353.4 cm^2 Ⓓ 447.7 cm^2

26. How many 2-inch squares will completely cover a rectangular prism 10 inches long, 4 inches wide, and 6 inches high?

Ⓐ 40 Ⓑ 62 Ⓒ 240 Ⓓ 248

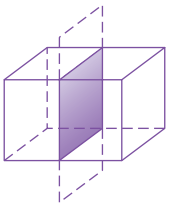
Extending the Lesson

If you make cuts in a solid, different 2-dimensional cross sections result, as shown at the right. Describe the cross section of each figure cut below.

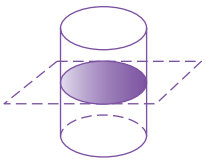


rectangle

27.



28.



29.

Maintain Your Skills

Mixed Review

Find the volume of each solid. If necessary, round to the nearest tenth. *(Lessons 11-2 and 11-3)*

30. rectangular pyramid: length 6 ft, width 5 ft, height 7 ft

31. cylinder: diameter 6 in., height 20 in.

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each product.**

*(To review **multiplying rational numbers**, see Lesson 5-3.)*

32. 10.3(8) **33.** 3.9(3.9) **34.** 12.3(9.2)(6)

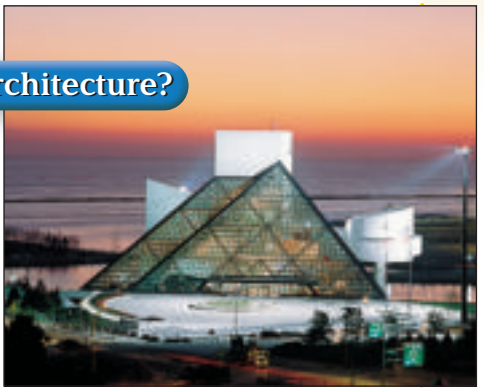
35. $\frac{1}{2} \cdot 2.6$ **36.** $\frac{1}{2} \cdot 82 \cdot 90$ **37.** $\frac{1}{2}\left(6\frac{1}{2}\right)$

11-5 Surface Area: Pyramids and Cones

What You'll Learn

- Find surface areas of pyramids.
- Find surface areas of cones.

How is surface area important in architecture?



The front of the Rock and Roll Hall of Fame in Cleveland, Ohio, is a glass pyramid.

a. The front triangle has a base of about 230 feet and height of about 120 feet. What is the area?

b. How could you find the total amount of glass used in the pyramid?

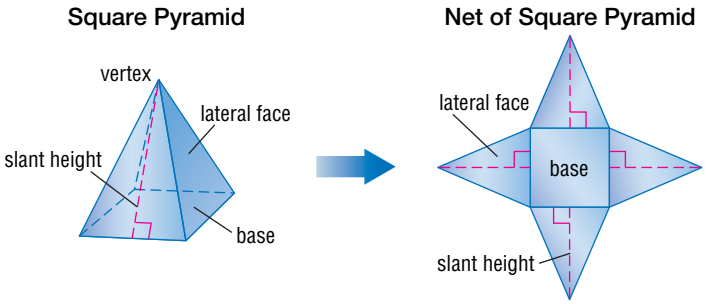
Vocabulary

- lateral face
- slant height
- lateral area

California Standards

Standard 7MG2.1 Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the **surface area** and volume of **basic three-dimensional figures**, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders. (CAHSEE)

SURFACE AREAS OF PYRAMIDS The sides of a pyramid are called **lateral faces**. They are triangles that intersect at the vertex. The altitude or height of each lateral face is called the **slant height**.

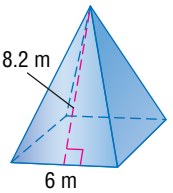


The sum of the areas of the lateral faces is the **lateral area** of a pyramid. The surface area of a pyramid is the lateral area plus the area of the base.

Example 1 Surface Area of a Pyramid

Find the surface area of the square pyramid.

Find the lateral area and the base area.



Area of each lateral face

$A = \frac{1}{2}bh$ Area of a triangle

$A = \frac{1}{2}(6)(8.2)$ Replace b with 6 and h with 8.2.

$A = 24.6$ Simplify.

There are 4 faces, so the lateral area is 4(24.6) or 98.4 square meters.

Area of base

$A = s^2$ Area of a square

$A = 6^2$ or 36 Replace s with 6 and simplify.

The surface area of a pyramid	equals	the lateral area	plus	the area of the base.
S	$=$	98.4	$+$	36

The surface area of the pyramid is 134.4 square meters.

Example 2 Use Surface Area to Solve a Problem

ARCHITECTURE **The Louvre museum in Paris has a huge square glass pyramid at the entrance with a slant height of about 92 feet. Its square base is 116 feet on each side. How much glass did it take to cover the pyramid?**

Find the lateral area only, since the bottom of the pyramid is not covered in glass.

$A = \frac{1}{2}bh$ Formula for area of a triangle

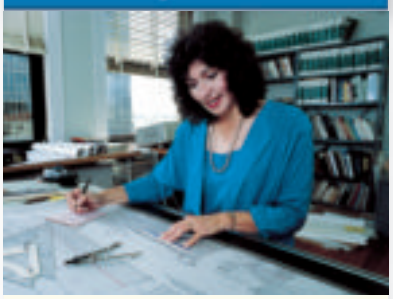
$A = \frac{1}{2}(116)(92)$ Replace b with 116 and h with 92.

$A = 5336$ Simplify.

One lateral face has an area of 5336 square feet. There are 4 lateral faces, so the lateral area is $4 \cdot 5336$ or 21,344 square feet.

It took 21,344 square feet of glass to cover the pyramid.

Career Choices

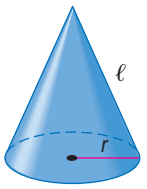


Architect
Architects must be able to combine geometric solids and understand their properties to form unified structures.

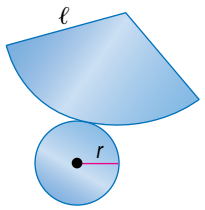
Online Research
For information about a career as an architect, visit: www.pre-alg.com/careers

SURFACE AREAS OF CONES You can also find surface areas of cones. The net of a cone shows the regions that make up the cone.

Model of Cone



Net of Cone

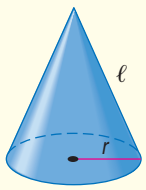


The lateral area of a cone with slant height ℓ is one-half the circumference of the base, $2\pi r$, times ℓ. So $A = \frac{1}{2} \cdot 2\pi r \cdot \ell$ or $A = \pi r\ell$. The base of the cone is a circle with area πr^2.

The surface area of a cone	equals	the lateral area	plus	the area of the base.
S	$=$	$\pi r\ell$	$+$	πr^2

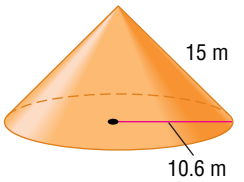
Key Concept — Surface Area of a Cone

- **Words** The surface area S of a cone with slant height ℓ and radius r is the lateral area plus the area of the base.
- **Symbols** $S = \pi r\ell + \pi r^2$
- **Model**



Example 3 Surface Area of a Cone

Find the surface area of the cone. Round to the nearest tenth.



$S = \pi r\ell + \pi r^2$ — Formula for surface area of a cone

$S = \pi(10.6)(15) + \pi(10.6)^2$ — Replace r with 10.6 and ℓ with 15.

$S \approx 852.5$ — Simplify.

The surface area of the cone is about 852.5 square meters.

Concept Check What is the formula for the lateral area L of a cone?

Check for Understanding

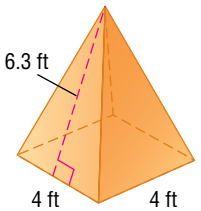
Concept Check

1. **Describe** the difference between slant height and height of a pyramid and a cone.
2. **Explain** how to find the lateral area of a pyramid.
3. **OPEN ENDED** Describe a situation in everyday life when a person might use the formulas for the surface area of a cone or a pyramid.

Guided Practice

Find the surface area of each solid. If necessary, round to the nearest tenth.

4.



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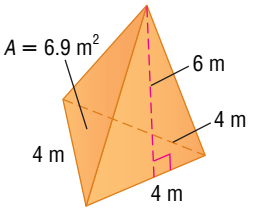

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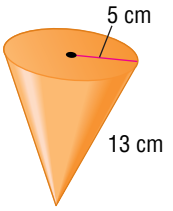
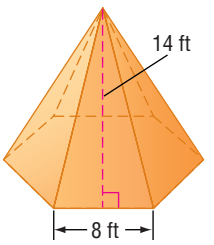


Application

7. **ARCHITECTURE** The small tower of a historic house is shaped like a regular hexagonal pyramid as shown at the right. How much roofing will be needed to cover this tower? (*Hint:* Do not include the base of the pyramid.)



Practice and Apply

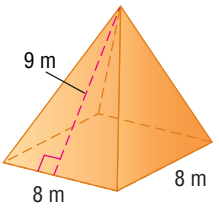
Homework Help

For Exercises	See Examples
8–11, 15, 16	1
12–14	3
17–19	2

Extra Practice See page 751.

Find the surface area of each solid. If necessary, round to the nearest tenth.

8.



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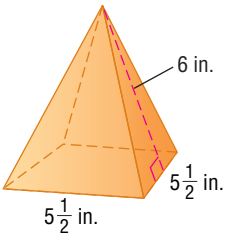

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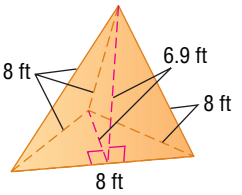
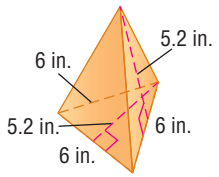


Study Tip

Triangular Pyramids
Assume that when the base of a pyramid is an equilateral triangle, all the faces are congruent.

Find the surface area of each solid. If necessary, round to the nearest tenth.

11.



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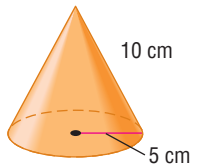

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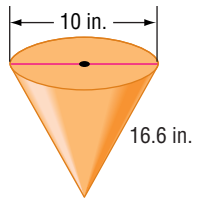

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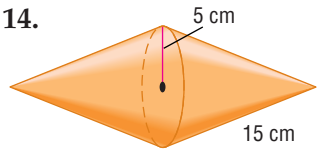

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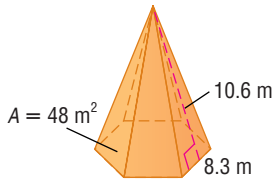

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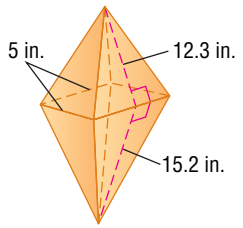
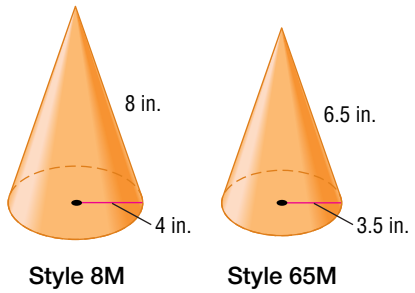


17. cone: radius 7.5 mm, slant height 14 mm

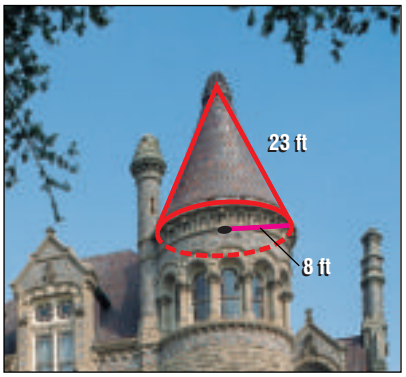
18. square pyramid: base side length 9 yd, slant height 8 yd

19. FUND-RAISING The cheerleaders are selling small megaphones decorated with the school mascot. There are two sizes, as shown at the right. What is the difference in the amount of plastic used in these two sizes? Round to the nearest square inch. (Note that a megaphone is open at the bottom.)



ARCHITECTURE **For Exercises 20 and 21, use the following information.** A roofing company is preparing bids on two special jobs involving cone-shaped roofs. Roofing material is usually sold in 100-square-foot squares. For each roof, find the lateral surface area to the nearest square foot. Then determine the squares of roofing materials that would be needed to cover each surface.

20.



21.

22. CRITICAL THINKING A bar of lead in the shape of a rectangular prism 13 inches by 2 inches by 1 inch is melted and recast into 100 conical fishing sinkers. The sinkers have a diameter of 1 inch, a height of 1 inch, and a slant height of about 1.1 inches. Compare the total surface area of all the sinkers to the surface area of the original lead bar.

23. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is surface area important in architecture?

Include the following in your answer:

- examples of how surface area is used in architecture, and
- an explanation of why building contractors and architects need to know surface areas.

Standards Practice

Standardized Test Practice

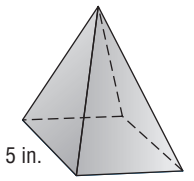
24. Find the surface area of a cone with a radius of 7 centimeters and slant height of 11.4 centimeters.

Ⓐ 153.9 cm^2 Ⓑ 250.7 cm^2 Ⓒ 272.7 cm^2 Ⓓ 404.6 cm^2

25. What is the lateral area of the square pyramid at the right if the slant height is 7 inches?

Ⓐ 17.5 in^2 Ⓑ 35 in^2

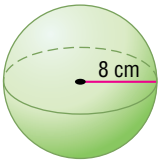
Ⓒ 70 in^2 Ⓓ 95 in^2



Extending the Lesson

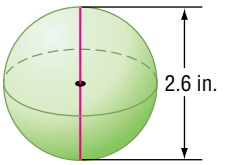
The surface area S of a sphere with radius r is given by the formula $S = 4\pi r^2$. Find the surface area of each sphere to the nearest tenth.

26.



27.

28.



Maintain Your Skills

Mixed Review

Find the surface area of each solid. If necessary, round to the nearest tenth. *(Lesson 11-4)*

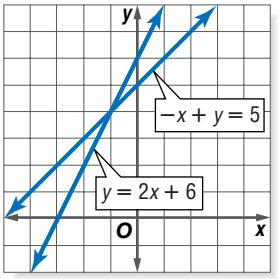
29. rectangular prism: length 2 ft, width 1 ft, height 0.5 ft

30. cylinder: radius 4 cm, height 13.8 cm

31. Find the volume of a cone that has a height of 6 inches and radius of 2 inches. Round to the nearest tenth. *(Lesson 11-3)*

State the solution of each system of equations. *(Lesson 8-9)*

32.



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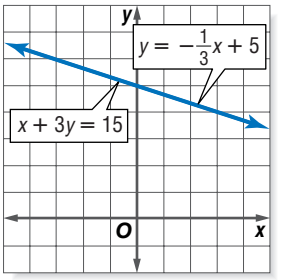


Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each proportion.**

*(To review **proportions**, see Lesson 6-2.)*

34. $\frac{1}{6} = \frac{x}{24}$

35. $\frac{9}{15} = \frac{n}{5}$

36. $\frac{t}{7} = \frac{40}{56}$

37. $\frac{1.6}{y} = \frac{9.6}{18}$

38. $\frac{4}{3.2} = \frac{w}{20}$

39. $\frac{18}{21} = \frac{2.7}{n}$

Geometry Activity

A Preview of Lesson 11-6

California Standards Standard 7MG2.3 Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor. (CAHSEE)

Similar Solids

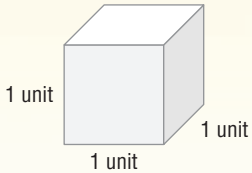
A model car is an exact replica of a real car, but much smaller. The dimensions of the model and the original are proportional. Therefore, these two objects are *similar solids*. The number of times that you increase or decrease the linear dimensions is called the scale factor.

You can use sugar cubes or centimeter blocks to investigate similar solids.

Activity 1

Collect the Data

- If each edge of a sugar cube is 1 unit long, then each face is 1 square unit and the volume of the cube is 1 cubic unit.
- Make a cube that has sides twice as long as the original cube.



Analyze the Data

1. How many small cubes did you use?
2. What is the area of one face of the original cube?
3. What is the area of one face of the cube that you built?
4. What is the volume of the original cube?
5. What is the volume of the cube that you built?

Activity 2

Collect the Data

Build a cube that has sides three times longer than a sugar cube or centimeter block.

Analyze the Data

6. How many small cubes did you use?
7. What is the area of one face of the cube?
8. What is the volume of the cube?
9. Complete the table at the right.
10. What happens to the area of a face when the length of a side is doubled? tripled?
11. Considering the unit cube, if the scale factor is x, what is the area of one face? the surface area?

Scale Factor	Side Length	Perimeter of a Face	Area of a Face	Volume
1				
2				
3				

12. What happens to the volume of a cube when the length of a side is doubled? tripled?
13. Considering the unit cube, let the scale factor be x. Write an expression for the cube's volume.
14. **Make a conjecture** about the surface area and the volume of a cube if the sides are 4 times longer than the original cube.
15. **RESEARCH** the scale factor of a model car. Use the scale factor to estimate the surface area and volume of the actual car.

11-6 Similar Solids

What You'll Learn

- Identify similar solids.
- Solve problems involving similar solids.

Vocabulary

- similar solids

California Standards

Standard 7MG2.3 Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. **Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor.** (CAHSEE)

Standard 7MR3.1 Evaluate the reasonableness of the solution in the context of the original situation.

How can linear dimensions be used to identify similar solids?

The model train below is $\frac{1}{87}$ the size of the original train.

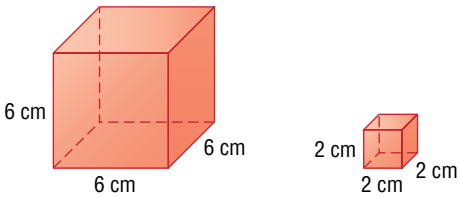
a. The model boxcar is shaped like a rectangular prism. If it is 8.5 inches long and 1 inch wide, what are the length and width of the original train boxcar to the nearest hundredth of a foot?

b. A model tank car is 7 inches long and is shaped like a cylinder. What is the length of the original tank car?

c. **Make a conjecture** about the radius of the original tank car compared to the model.

IDENTIFY SIMILAR SOLIDS

The cubes below have the same shape. The ratio of their corresponding edge lengths is $\frac{6}{2}$ or 3. We say that 3 is the scale factor.



The cubes are **similar solids** because they have the same shape and their corresponding linear measures are proportional.

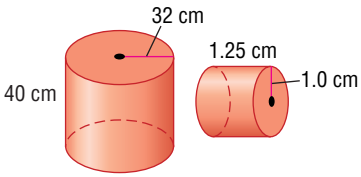
Study Tip

Look Back
To review **similar figures**, see Lesson 6-3.

Example 1 Identify Similar Solids

Determine whether each pair of solids is similar.

a.



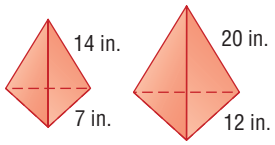
$\frac{32}{1} \stackrel{?}{=} \frac{40}{1.25}$ Write a proportion comparing radii and heights.

$32(1.25) \stackrel{?}{=} 1(40)$ Find the cross products.

$40 = 40$ ✓ Simplify.

The radii and heights are proportional, so the cylinders are similar.

b.



$$\frac{14}{20} \stackrel{?}{=} \frac{7}{12}$$ Write a proportion comparing corresponding edge lengths.

$$14(12) \stackrel{?}{=} 20(7)$$ Find the cross products.

$$168 \neq 140$$ Simplify.

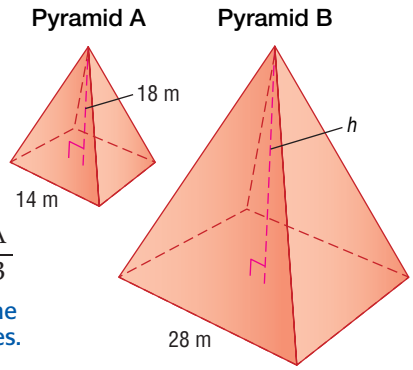
The corresponding measures are not proportional, so the pyramids are not similar.

USE SIMILAR SOLIDS

You can find missing measures if you know solids are similar.

Example 2 Find Missing Measures

The square pyramids at the right are similar. Find the height of pyramid B.



$$\frac{\text{base length of pyramid A}}{\text{base length of pyramid B}} = \frac{\text{height of pyramid A}}{\text{height of pyramid B}}$$

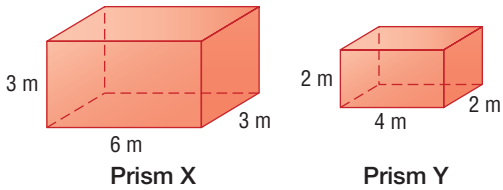
$$\frac{14}{28} = \frac{18}{h}$$ Substitute the known values.

$$14h = 28(18)$$ Find the cross products.

$$h = 36$$ Simplify.

The height of pyramid B is 36 meters.

The prisms at the right are similar with a scale factor of $\frac{3}{2}$.



Prism	Surface Area	Volume
X	90 m^2	54 m^3
Y	40 m^2	16 m^3

Study Tip

Scale Factors
When the lengths of all dimensions of a solid are multiplied by a scale factor x, then the surface area is multiplied by x^2 and the volume is multiplied by x^3.

Notice the pattern in the following ratios.

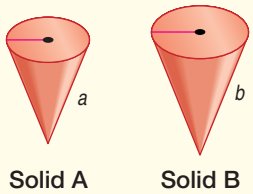
$$\frac{\text{surface area of prism X}}{\text{surface area of prism Y}} = \frac{90}{40} \text{ or } \frac{9}{4} \quad \Rightarrow \quad \frac{9}{4} = \frac{3^2}{2^2}$$

$$\frac{\text{volume of prism X}}{\text{volume of prism Y}} = \frac{54}{16} \text{ or } \frac{27}{8} \quad \Rightarrow \quad \frac{27}{8} = \frac{3^3}{2^3}$$

This and other similar examples suggest that the following ratios are true for similar solids.

Key Concept — Ratios of Similar Solids

- **Words** If two solids are similar with a scale factor of $\frac{a}{b}$, then the surface areas have a ratio of $\frac{a^2}{b^2}$ and the volumes have a ratio of $\frac{a^3}{b^3}$.
- **Model**



Example 3 Use Similar Solids to Solve a Problem

SPACE TRAVEL **A scale model of the NASA space capsule is a combination of a truncated cone and cylinder. The small model built by engineers on a scale of 1 cm to 20 cm has a volume of 155 cm³. What is the volume of the actual space capsule?**

More About . . .

Space

Engineers need to know the volume of the actual space capsules in order to estimate the air pressure.

Source: www.space.about.com

Explore You know the scale factor $\frac{a}{b}$ is $\frac{1}{20}$ and the volume of the space capsule is 155 cm³.

Plan Since the volumes have a ratio of $\frac{a^3}{b^3}$ and $\frac{a}{b} = \frac{1}{20}$, replace a with 1 and b with 20 in $\frac{a^3}{b^3}$.

Solve

$$\frac{\text{volume of model}}{\text{volume of capsule}} = \frac{a^3}{b^3} \quad \text{Write the ratio of volumes.}$$

$$= \frac{1^3}{20^3} \quad \text{Replace } a \text{ with 1 and } b \text{ with 20.}$$

$$= \frac{1}{8000} \quad \text{Simplify.}$$

So, the volume of the capsule is 8000 times the volume of the model.

$8000 \cdot 155 \text{ cm}^3 = 1{,}240{,}000 \text{ cm}^3$

Examine Use estimation to check the reasonableness of this answer. $8000 \cdot 100 = 800{,}000$ and $8000 \cdot 200 = 1{,}600{,}000$, so the answer must be between 800,000 and 1,600,000. The answer 1,240,000 cm³ is reasonable.

Check for Understanding

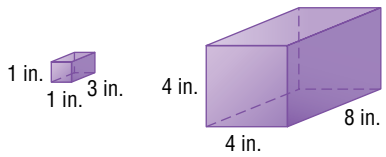
Concept Check

1. **OPEN ENDED** Draw and label two cones that are similar. Explain why they are similar.
2. **Explain** how you can find the surface area of a larger cylinder if you know the surface area of a smaller cylinder that is similar to it and the scale factor.

Guided Practice

Determine whether each pair of solids is similar.

3.



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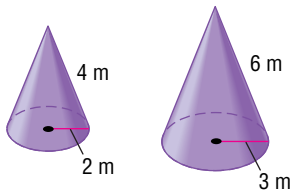
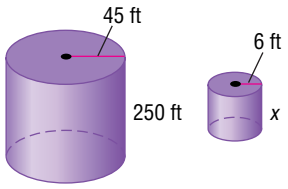


Find the missing measure for each pair of similar solids.

5.



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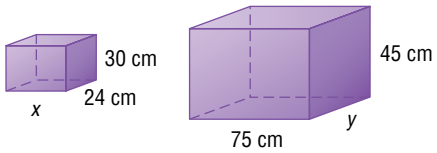


Application **ARCHITECTURE** **For Exercises 7–9, use the following information.**
A model for an office building is 60 centimeters long, 42 centimeters wide, and 350 centimeters high. On the model, 1 centimeter represents 1.5 meters.

7. How tall is the actual building in meters?

8. What is the scale factor between the model and the building?

9. Determine the volume of the building in cubic meters.

Practice and Apply

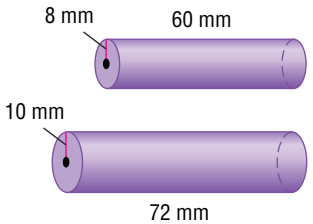
Homework Help

For Exercises	See Examples
10–13, 16–19	1
14, 15	2
20, 21	3

Extra Practice
See page 752.

Determine whether each pair of solids is similar.

10.



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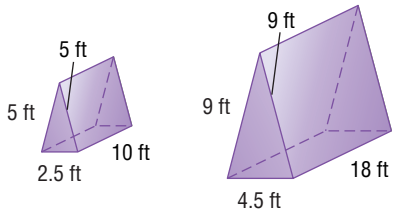

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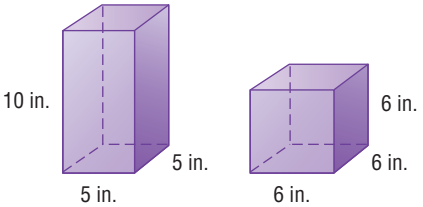

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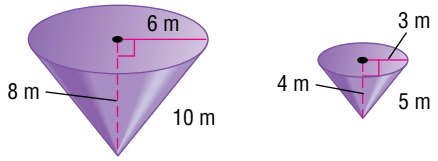
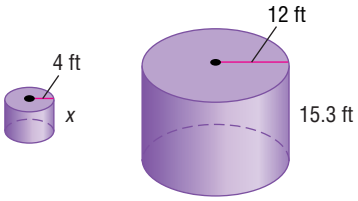


Find the missing measure for each pair of similar solids.

14.



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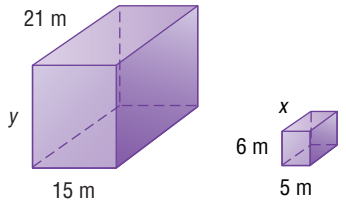


Determine whether each pair of solids is *sometimes*, *always*, or *never* similar. Explain.

16. two cubes

17. two prisms

18. a cone and a cylinder

19. two spheres

HISTORY **The *Mankaure* pyramid in Egypt has a square base that is 110 meters on each side, a height of 68.8 meters, and a slant height of 88.5 meters. Suppose you want to construct a scale model of the pyramid using a scale of 4 meters to 2 centimeters.**

20. How much material will you need to use?

21. How much greater is the volume of the actual pyramid to the volume of the model?

Online Research **Data Update** How have the dimensions of the Egyptian pyramids changed in thousands of years? Visit www.pre-alg.com/data_update to learn more.

22. **CRITICAL THINKING** The dimensions of a triangular prism are decreased so that the volume of the new prism is $\frac{1}{3}$ that of the original volume. Are the two prisms similar? Explain.

23. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can linear dimensions be used to identify similar solids?

Include the following in your answer:

- a description of the ratios needed for two solids to be similar, and
- an example of two solids that are *not* similar.

Standardized Test Practice

24. Which prism shown in the table is *not* similar to the other three?

Ⓐ prism A Ⓑ prism B

Ⓒ prism C Ⓓ prism D

Prism	Length	Width	Height
A	4	3	2
B	6	4.5	3
C	5	4	2
D	28	21	14

25. If the dimensions of a cone are doubled, the surface area

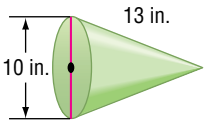
Ⓐ stays the same. Ⓑ is doubled.

Ⓒ is quadrupled. Ⓓ is 8 times greater.

Maintain Your Skills

Mixed Review

Find the surface area of each solid. If necessary, round to the nearest tenth. *(Lessons 11-4 and 11-5)*

26.


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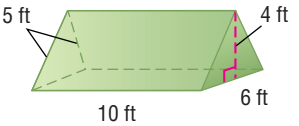

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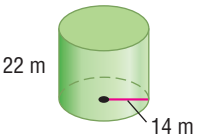


29. Angles J and K are complementary. Find $m\angle K$ if $m\angle J$ is 25°. *(Lesson 10-2)*

Solve each equation. Check your solution. *(Lesson 5-9)*

30. $r - 3.5 = 8$ 31. $\frac{2}{3} + y = \frac{1}{9}$ 32. $\frac{1}{4}a = 6$

Getting Ready for the Next Lesson

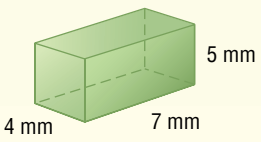
PREREQUISITE SKILL **Find the value of each expression to the nearest tenth.** *(To review **rounding decimals**, see page 711.)*

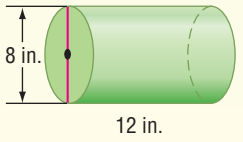
33. $13.28 + 6.05$ 34. $8.99 - 1.2$ 35. $2.4 \cdot 2.5$

36. $55 \div 3.8$ 37. $6 + 1.9 + 1.45$ 38. $6.7(0.3)(1.8)$

Practice Quiz 2 — Lessons 11-4 through 11-6

Find the surface area of each solid. If necessary, round to the nearest tenth. *(Lessons 11-4 and 11-5)*

1.


2.


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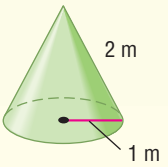

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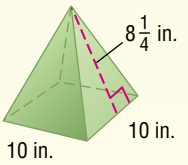


5. Are the cylinders described in the table similar? Explain your reasoning. *(Lesson 11-6)*

Cylinder	Diameter (mm)	Slant Height (mm)
A	24	21
B	16	14

Precision and Accuracy

In everyday language, *precision* and *accuracy* are used to mean the same thing. When measurement is involved, these two terms have different meanings.

Term	Definition	Example
precision	the degree of exactness in which a measurement is made	A measure of 12.355 grams is more precise than a measure of 12 grams.
accuracy	the degree of conformity of a measurement with the true value	Suppose the actual mass of an object is 12.355 grams. Then a measure of 12 grams is more accurate than a measure of 18 grams.

Reading to Learn

1. Describe in your own words the difference between accuracy and precision.

2. **RESEARCH** Use the Internet or other resources to find an instrument used in science that gives very precise measurements. Describe the precision of the instrument.

3. Use at least two different measuring instruments to measure the length, width, height, or weight of two objects in your home. Describe the measuring instruments that you used and explain which measurement was most precise.

Choose the correct term or terms to determine the degree of precision needed in each measurement situation.

4. In a travel brochure, the length of a cruise ship is described in (millimeters, meters).

5. The weight of a bag of apples in a grocery store is given to the nearest (tenth of a pound, tenth of an ounce).

6. In a science experiment, the mass of one drop of solution is found to the nearest 0.01 (gram, kilogram).

7. A person making a jacket measures the fabric to the nearest (inch, eighth of an inch).

8. **CONSTRUCTION** A construction c ompany is ordering cement to complete all the sidewalks in a new neighborhood. Would the precision or accuracy be more important in the completion of their order? Explain.

11-7 Precision and Significant Digits

What You'll Learn

- Describe measurements using precision and significant digits.
- Apply precision and significant digits in problem-solving situations.

Vocabulary

- precision
- significant digits

California Standards
Standard 7MR2.7
Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

Why are all measurements really approximations?

Use cardboard to make three rulers 20 centimeters long, labeling the increments as shown at the right.

Ruler	Scale (cm)
1	0, 5, 10, 15, 20
2	0, 1, 2, 3, …, 18, 19, 20
3	0, 0.1, 0.2, 0.3, …, 19.8, 19.9, 20.0

a. Measure several objects (book widths, paper clips, pens…) using each ruler. Use a table to keep track of your measurements.

b. Analyze the measurements and determine which are most useful. Explain your reasoning.

PRECISION AND SIGNIFICANT DIGITS

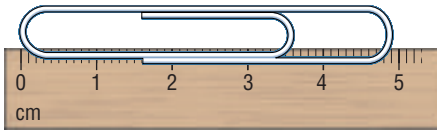
The **precision** of a measurement is the exactness to which a measurement is made. Precision depends on the smallest unit of measure being used, or the *precision unit*. You can expect a measurement to be accurate to the nearest precision unit.

The precision unit of this ruler is 1 centimeter.

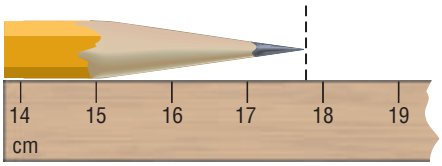
Example 1 Identify Precision Units

Identify the precision unit of the ruler at the right.

The precision unit is one tenth of a centimeter, or 1 millimeter.



One way to record a measure is to estimate to the nearest precision unit. A more precise method is to include all of the digits that are actually measured, plus one *estimated* digit. The digits you record when you measure this way are called significant digits. **Significant digits** indicate the precision of the measurement.



estimated digit

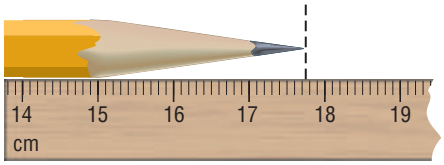
17.7 cm ← 3 significant digits

digits known for certain

precision unit: 1 cm
actual measure: 17–18 cm
estimated measure: 17.7 cm

Study Tip

Precision
The precision unit of the measuring instrument determines the number of significant digits.



estimated digit

17.75 cm ← 4 significant digits

digits known for certain

precision unit: 0.1 cm
actual measure: 17.7–17.8 cm
estimated measure: 17.75 cm

There are special rules for determining significant digits in a given measurement. If a number contains a decimal point, the number of significant digits is found by counting the digits from left to right, starting with the first *nonzero* digit and ending with the last digit.

Number		Number of Significant Digits	
1.23	→	3	All nonzero digits are significant.
10.05	→	4	Zeros between two significant digits are significant.
0.072	→	2	Zeros used to show place value of the decimal are not significant.
50.00	→	4	In a number with a decimal point, all zeros to the right of a nonzero digit are significant.

If a number does *not* contain a decimal point, the number of significant digits is found by counting the digits from left to right, starting with the first digit and ending with the last *nonzero* digit. For example, 8400 contains 2 significant digits, 8 and 4.

Study Tip

Precision Units
Since 150 miles has only two significant digits, the measure is precise to the nearest 10 miles. Therefore, the precision unit is 10 miles, not 1 mile.

Example 2 Identify Significant Digits

Determine the number of significant digits in each measure.

a. 20.98 centimeters
4 significant digits

b. 150 miles
2 significant digits

c. 0.007 gram
1 significant digit

d. 6.40 feet
3 significant digits

COMPUTE USING SIGNIFICANT DIGITS When adding or subtracting measurements, the sum or difference should have the *same precision* as the least precise measurement.

Example 3 Add Measurements

The sides of a triangle measure 14.35 meters, 8.6 meters, and 9.125 meters. Use the correct number of significant digits to find the perimeter.

$$\begin{array}{rl} 14.35 & \leftarrow \text{2 decimal places} \\ 8.6 & \leftarrow \text{1 decimal place} \\ +\ 9.125 & \leftarrow \text{3 decimal places} \\ \hline 32.075 & \end{array}$$

The least precise measurement, 8.6 meters, has one decimal place. So, round 32.075 to one decimal place, 32.1. The perimeter of the triangle is about 32.1 meters.

Study Tip

Common Misconception
Calculators give answers with as many digits as the display can show. Be sure to use the correct number of significant digits in your answer.

When multiplying or dividing measurements, the product or quotient should have the *same number of significant digits* as the measurement with the least number of significant digits.

Study Tip

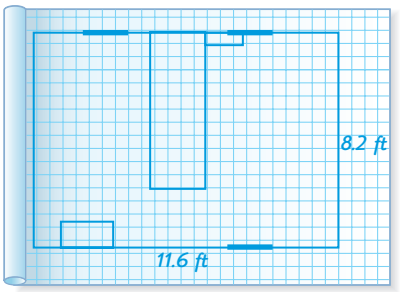
Significant Digits
The least precise measure determines the number of significant digits in the sum or difference of measures. The measure with the fewest significant digits determines the number of significant digits in the product or quotient of measures.

Example 4 Multiply Measurements

What is the area of the bedroom shown at the right?

To find the area, multiply the length and the width.

$$\begin{array}{rl} 11.6 & \leftarrow \text{3 significant digits} \\ \times\ 8.2 & \leftarrow \text{2 significant digits} \\ \hline 95.12 & \leftarrow \text{4 significant digits} \end{array}$$



The answer cannot have more significant digits than the measurements of the length and width. So, round 95.12 ft^2 to 2 significant digits. The area of the bedroom is about 95 ft^2.

Check for Understanding

Concept Check

1. **FIND THE ERROR** A metal shelf is 0.0205 centimeter thick. Sierra says this measurement contains 2 significant digits. Josh says it contains 3 significant digits. Who is correct? Explain your reasoning.

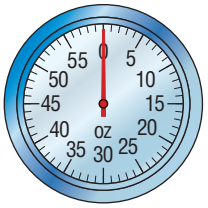
2. **Choose** an instrument from the list at the right that is best for measuring each object.
   a. length of an envelope
   b. distance between two stoplights
   c. width of a kitchen
   d. height of a small child

yardstick
centimeter ruler
surveyor's tools
12-foot tape measure

3. **OPEN ENDED** Write a number that contains four digits, two of which are significant.

Guided Practice

4. Identify the precision unit of the scale at the right.



Determine the number of significant digits in each measure.

5. 2.30 cm
6. 50 yd
7. 0.801 mm

Calculate. Round to the correct number of significant digits.

8. 14.38 cm + 5.7 cm
9. 15.273 L − 8.2 L
10. 3.147 mm · 1.8 mm
11. 60.42 in. × 9.012 in.

Application

12. **MASONRY** A wall of bricks is 7.85 feet high and 13.0 feet wide. What is the area of the wall? Round to the correct number of significant digits.

Practice and Apply

Homework Help

For Exercises	See Examples
13, 14	1
15–22	2
23–38	3, 4

Extra Practice
See page 752.

Identify the precision unit of each measuring tool.

13. 

14. 

Determine the number of significant digits in each measure.

15. 925 g **16.** 40 km **17.** 2200 ft **18.** 53.6 in.

19. 0.01 mm **20.** 0.56 cm **21.** 18.50 m **22.** 4.0 L

Calculate. Round to the correct number of significant digits.

23. 27 in. + 18.2 in.

24. 6.75 mm – 3.2 mm

25. 0.4 ft · 5.1 ft

26. 7.30 yd × 1.61 yd

27. 29.307 m + 4.23 m + 50.93 m

28. 127.2 g + 42.3 g – 5.7 g

29. 50.2 cm – 0.75 cm

30. 18.160 L – 15 L

31. 5.327 m · 4.8 m

32. 4.397 cm · 2.01 cm

33. **MEASUREMENT** Choose the best ruler for measuring an object to the nearest sixteenth of an inch. Explain your reasoning.

a. 

b. 

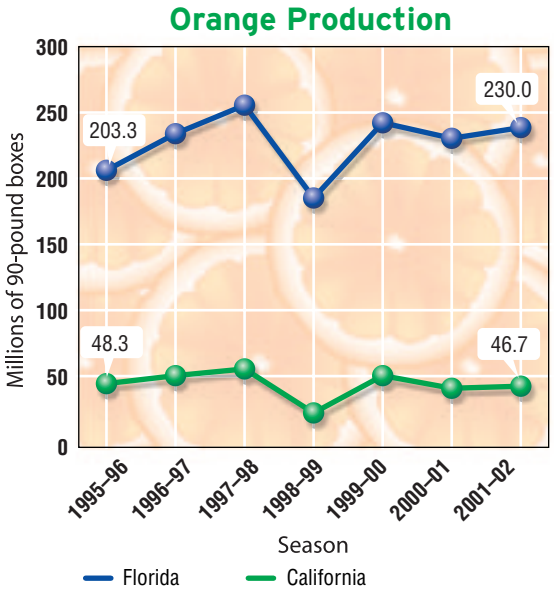
34. **MEASUREMENT** Order 0.40 mm, 40 mm, 0.4 mm, and 0.004 mm from most to least precise.

ORANGES **For Exercises 35–37, refer to the graph at the right.**

35. Are the numbers exact? Explain.

36. How many significant digits are used to describe orange production in the 1995–96 season in Florida and California?

37. Write the number of millions of 90-pound boxes of oranges produced in California in the 2001–02 season without using a decimal point. How many significant digits does this number have?



Source: Florida Department of Citrus

38. **MEASUREMENT ERROR** Mrs. Hernandez is covering the top of a kitchen shelf that is $26\frac{3}{8}$ inches long and $15\frac{1}{2}$ inches wide. She incorrectly measures the length to be 25 inches long. How will this error affect her calculations?

39. **CRITICAL THINKING** The sizes of Allen hex wrenches are 2.0 mm, 3.0 mm, 4.0 mm, and so on. Will they work with hexagonal bolts that are marked 2 mm, 3 mm, 4 mm, and so on? Explain.

40. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

Why are all measurements really approximations?

Include the following in your answer:

- an explanation of what determines the precision of a measurement, and
- an example of a real-life situation in which an exact solution is needed and a situation in which an approximate solution is sufficient.

Standards Practice

Standardized Test Practice

41. Choose the measurement that is most precise.

Ⓐ 12 mm Ⓑ 12 cm Ⓒ 1.2 m Ⓓ 12 m

42. Which solution contains the correct number of significant digits for the product 2.80 mm · 0.1 mm?

Ⓐ 0.28 mm^2 Ⓑ 0.280 mm^2 Ⓒ 0.30 mm^2 Ⓓ 0.3 mm^2

Extending the Lesson

The **greatest possible error** is one-half the precision unit. It can be used to describe the actual measure. Refer to the paper clip in Example 1. It appears to be 4.9 centimeters long.

$$\text{greatest possible error} = \frac{1}{2} \cdot \text{precision unit}$$
$$= \frac{1}{2} \cdot 0.1 \text{ cm or } 0.05 \text{ cm}$$

The possible actual length of the paper clip is 0.05 centimeter less than or 0.05 centimeter greater than 4.9 centimeters, or between 4.85 and 4.95 centimeters.

43. **TRAVEL** The odometer on a car shows 132.8 miles traveled. Find the greatest possible error of the measurement and use it to determine between which two values is the actual distance traveled.

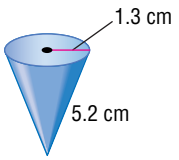
Maintain Your Skills

Mixed Review

44. Determine whether a cone with a height 14 centimeters and radius 8 centimeters is similar to a cone with a height of 12 centimeters and a radius of 6 centimeters. *(Lesson 11-6)*

Find the surface area of each solid. If necessary, round to the nearest tenth. *(Lesson 11-5)*

45.



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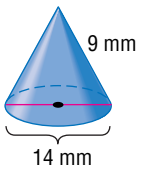

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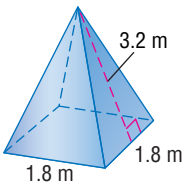


WebQuest Internet Project

Able to Leap Tall Buildings

It is time to complete your project. Use the information you have gathered about your building to prepare a report. Be sure to include information and facts about your building as well as a comparison of its size to some familiar item.

www.pre-alg.com/webquest

Chapter 11

Study Guide and Review

Vocabulary and Concept Check

base (p. 557)
cone (p. 569)
cylinder (p. 565)
edge (p. 556)
face (p. 556)
lateral area (p. 578)
lateral face (p. 578)
plane (p. 556)
polyhedron (p. 556)
precision (p. 590)
prism (p. 557)
pyramid (p. 557)
significant digits (p. 590)
similar solids (p. 584)
skew lines (p. 558)
slant height (p. 578)
solid (p. 556)
surface area (p. 573)
vertex (p. 556)
volume (p. 563)

Determine whether each statement is *true* or *false*. If false, replace the underlined word or number to make a true statement.

1. The surface area of a pyramid is the sum of the areas of its lateral faces.
2. Volume is the amount of space that a solid contains.
3. The edge of a pyramid is the length of an altitude of one of its lateral faces.
4. A triangular prism has two bases.
5. A solid with two bases that are parallel circles is called a cone.
6. Prisms and pyramids are named by the shapes of their bases.
7. Figures that have the same shape and corresponding linear measures that are proportional are called similar solids.
8. Significant digits indicate the precision of a measurement.

Lesson-by-Lesson Review

11-1 Three-Dimensional Figures

See pages 556–561.

Concept Summary

- Prisms and pyramids are three-dimensional figures.

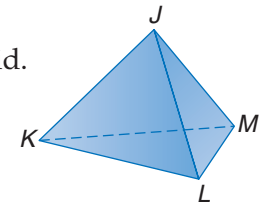
Example **Identify the solid. Name the bases, faces, edges, and vertices.**

There is one triangular base, so the solid is a triangular pyramid.

faces: *JKL*, *JLM*, *JMK*

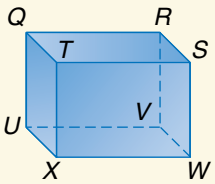
edges: $\overline{JK}$, $\overline{JL}$, $\overline{JM}$, $\overline{KL}$, $\overline{LM}$, $\overline{MK}$

vertices: *J*, *K*, *L*, *M*



Exercises **Identify each solid. Name the bases, faces, edges, and vertices.**

See Example 1 on page 557.

9.


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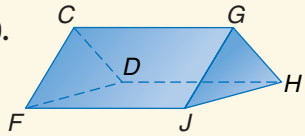
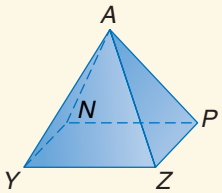


11.


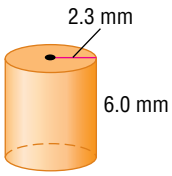
11-2 Volume: Prisms and Cylinders

See pages 563–567.

Concept Summary

- Volume is the measure of space occupied by a solid region.
- The volume of a prism or a cylinder is the area of the base times the height.

Example **Find the volume of the cylinder. Round to the nearest tenth.**



$V = \pi r^2 h$ Formula for volume of a cylinder

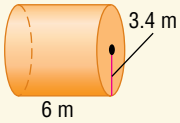
$V = \pi \cdot 2.3^2 \cdot 6.0$ Replace r with 2.3 and h with 6.0.

$V \approx 99.7$ Simplify.

The volume is about 99.7 cubic millimeters.

Exercises **Find the volume of each solid. If necessary, round to the nearest tenth.** *See Examples 1, 2, and 5 on pages 563–565.*

12.



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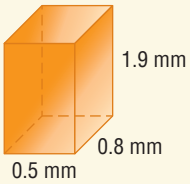

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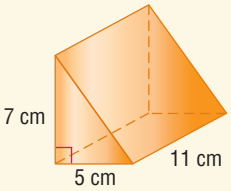


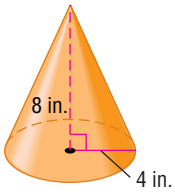
11-3 Volume: Pyramids and Cones

See pages 568–572.

Concept Summary

- The volume of a pyramid or a cone is one-third the area of the base times the height.

Example **Find the volume of the cone. Round to the nearest tenth.**



$V = \frac{1}{3}\pi r^2 h$ Formula for volume of a cone

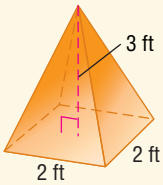
$V = \frac{1}{3} \cdot \pi \cdot 4^2 \cdot 8$ Replace r with 4 and h with 8.

$V \approx 134.0$ Simplify.

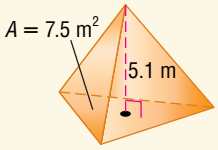
The volume is about 134.0 cubic inches.

Exercises **Find the volume of each solid. If necessary, round to the nearest tenth.** *See Examples 1 and 2 on pages 568 and 569.*

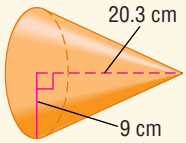
15.



16.



17.



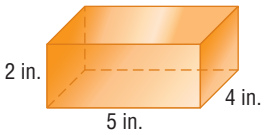
11-4 Surface Area: Prisms and Cylinders

See pages 573–577.

Concept Summary

- The surface area of a prism is the sum of the areas of the faces.
- The surface area of a cylinder is the area of the two bases plus the product of the circumference and the height.

Example **Find the surface area of the rectangular prism.**

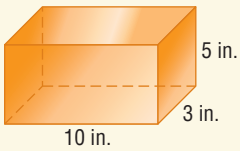


$S = 2\ell w + 2\ell h + 2wh$	Write the formula.
$S = 2(5)(4) + 2(5)(2) + 2(4)(2)$	Substitution
$S = 76$	Simplify.

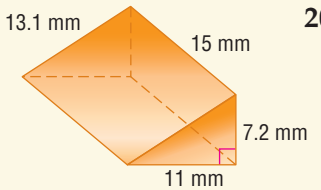
The surface area is 76 square inches.

Exercises **Find the surface area of each solid. If necessary, round to the nearest tenth.** *See Examples 1–3 on pages 573–575.*

18.



19.



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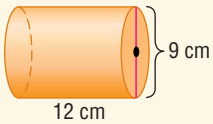


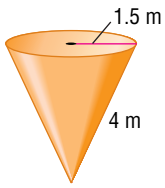
11-5 Surface Area: Pyramids and Cones

See pages 578–582.

Concept Summary

- The surface area of a pyramid or a cone is the sum of the lateral area and the base area.

Example **Find the surface area of the cone. Round to the nearest tenth.**

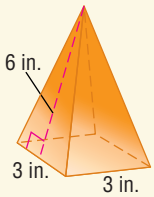


$S = \pi r\ell + \pi r^2$	Write the formula.
$S = \pi(1.5)(4) + \pi(1.5)^2$	Replace r with 1.5 and ℓ with 4.
$S \approx 25.9$	Simplify.

The surface area is about 25.9 square meters.

Exercises **Find the surface area of each solid. If necessary, round to the nearest tenth.** *See Examples 1 and 3 on pages 578 and 580.*

21.



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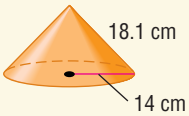

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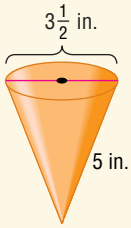


• Extra Practice, see pages 750–752.
• Mixed Problem Solving, see page 768.

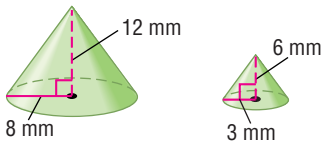
11-6 Similar Solids

See pages 584–588.

Concept Summary

- Similar solids have the same shape and their corresponding linear measures are proportional.

Example **Determine whether the solids are similar.**



$\frac{12}{6} \stackrel{?}{=} \frac{8}{3}$ Write a proportion comparing radii and heights.

$12(3) \stackrel{?}{=} 6(8)$ Find the cross products.

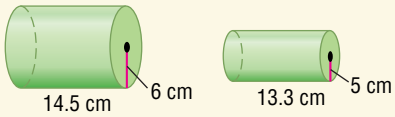
$36 \neq 48$ Simplify.

The radii and heights are not proportional, so the cones are not similar.

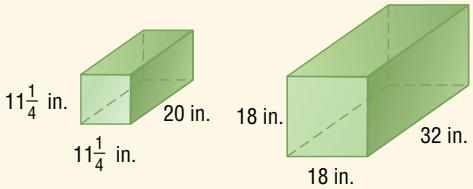
Exercises **Determine whether each pair of solids is similar.**

See Example 1 on pages 584 and 585.

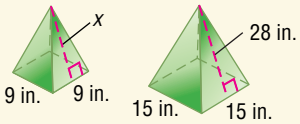
24.



25.



26. Find the missing measure for the pair of similar solids at the right.
See Example 2 on page 585.



11-7 Precision and Significant Digits

See pages 590–594.

Concept Summary

- The precision of a measurement depends on the smallest unit of measure being used.
- Significant digits indicate the precision of a measurement.

Example **Find 0.5 m + 0.75 m. Round to the correct number of significant digits.**

$$\begin{array}{r} 0.5 \\ +\ 0.75 \\ \hline 1.25 \end{array} \quad \begin{array}{l} \leftarrow \text{1 decimal place} \\ \leftarrow \text{2 decimal places} \\ \end{array}$$

The answer should have one decimal place. So, the sum is about 1.3 meters.

Calculate. Round to the correct number of significant digits.

See Examples 3 and 4 on pages 591 and 592.

27. 10.3 cm + 8.7 cm **28.** 25.71 kg − 11.2 kg **29.** 0.04 m + 0.9 m

30. 5.186 in. · 1.5 in. **31.** 32.0 ft · 30.4 ft **32.** 80.51 g − 6.01 g

Chapter 11 Practice Test

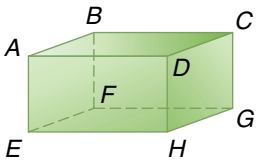
Vocabulary and Concepts

1. **Describe** the difference between a prism and a pyramid.
2. **Describe** the characteristics of similar solids.
3. **OPEN ENDED** Write a four-digit number that has three significant digits.

Skills and Applications

Identify each solid. Name the bases, faces, edges, and vertices.

4.



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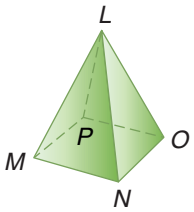
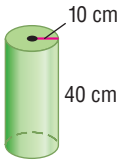


Find the volume of each solid. If necessary, round to the nearest tenth.

6. cylinder: radius 1.7 mm, height 8 mm
7. rectangular pyramid: length 14 in., width 8 in., height 5 in.
8. cube: length 9.2 cm
9. cone: diameter 26 ft, height 31 ft

Find the surface area of each solid. If necessary, round to the nearest tenth.

10.



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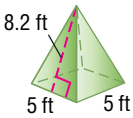

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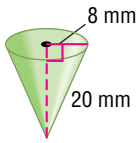
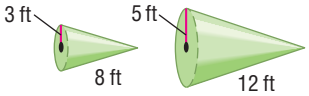
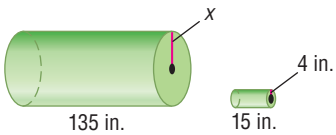


13. Determine whether the given pair of solids is similar. Explain.



Find the missing measure for each pair of similar solids.

14.



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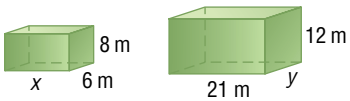


Determine the number of significant digits in each measure.

16. 4500 mm

17. 0.036 in.

Calculate. Round to the correct number of significant digits.

18. 37.65 cm – 12.9 cm

19. 6.8 ft × 3.875 ft

Standards Practice

20. **STANDARDIZED TEST PRACTICE** A model of a new grocery store is 15 inches long, 9 inches wide, and 7 inches high. The scale is 50 feet to 3 inches. Find the length of the actual store.

Ⓐ 45 ft Ⓑ 150 ft Ⓒ 250 ft Ⓓ 750 ft

Chapter 11 Standardized Test Practice

Standards Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. If $-3x + 7 = -29$, then what is the value of x? (Lesson 3-5)

Ⓐ -12 Ⓑ -6

Ⓒ 6 Ⓓ 12

2. Austin wants to buy a fish tank so that his fish get as much oxygen as possible. The pet shop has four different fish tanks. The dimensions below represent the length and width of each fish tank. Which tank has the greatest surface area at the top? (Lesson 3-7)

Ⓐ 22 in. × 18 in. Ⓑ 24 in. × 16 in.

Ⓒ 26 in. × 14 in. Ⓓ 28 in. × 12 in.

3. The Hyde family's weekly food expenses for four consecutive weeks were \$105.52, \$98.26, \$101.29, and \$91.73. What is the mean of their weekly food expenses for those four weeks? (Lesson 5-7)

Ⓐ \$98.63 Ⓑ \$98.75

Ⓒ \$99.20 Ⓓ \$99.78

4. Students taste-tested three brands of instant hot cereal and chose their favorite brand. Which of these statements is *not* supported by the data in the table? (Lesson 6-1)

	Hot Cereal Brand		
	X	Y	Z
Girls	12	7	10
Boys	10	15	5

Ⓐ Twice as many girls as boys chose Brand Z.

Ⓑ The total number of students who chose Brand X is equal to the total number who chose Brand Y.

Ⓒ Three times as many boys chose Brand Y as Brand Z.

Ⓓ Half of the students who chose Brand Z were boys.

5. A sloppy-joe recipe for 12 servings calls for 2 pounds of ground beef. How many pounds of ground beef will be needed to make 30 servings? (Lesson 6-3)

Ⓐ 2.5 lb Ⓑ 4.5 lb Ⓒ 5 lb Ⓓ 6 lb

6. Three-fourths of a county's population is registered to vote. Only 50% of the registered voters in the county actually voted in the election. What fractional part of the county's population voted? (Lesson 6-4)

Ⓐ $\frac{1}{2}$ Ⓑ $\frac{3}{8}$ Ⓒ $\frac{2}{3}$ Ⓓ $\frac{3}{4}$

7. Rod has \$10 to spend at an arcade. Each bag of popcorn at the arcade costs \$3.25, and each video game costs \$1.00. Which expression represents the amount of money Rod will have left after he buys one bag of popcorn and plays n video games? (Lesson 7-2)

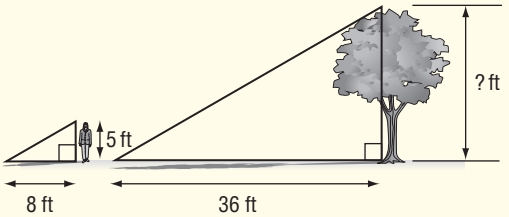
Ⓐ $10.00 + 3.25 + 1.00n$

Ⓑ $10.00 - 3.25 - 1.00n$

Ⓒ $3.25 - 1.00n - 10.00$

Ⓓ $10.00 - 3.25n - 1.00n$

8. Cheyenne is 5 feet tall. She measures her shadow and a tree's shadow at the same time of day, as shown in the diagram below. How tall is the tree? (Lesson 9-7)



Ⓐ 20 ft Ⓑ 22.5 ft Ⓒ 40 ft Ⓓ 57.6 ft

Test-Taking Tip

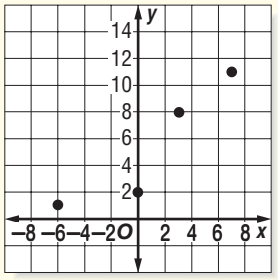
Pace yourself. Do not spend too much time on any one question. If you're having difficulty answering a question, mark it in your test booklet and go on to the next question. Make sure that you also skip the question on your answer sheet. At the end of the test, go back and answer the questions that you skipped.

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 771–788.

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. Determine the range of the relation shown in the graph. (Lesson 1-6)



10. Simplify $-9s(4t)$. (Lesson 2-4)

11. Thang is enclosing a rectangular area for his dog. He bought enough wire fencing to enclose 308 square feet of space. If he makes the length of the rectangle 22 feet, what is the width in feet? (Lesson 3-7)

12. What is the value of x^{-2} for $x = 3$? (Lesson 4-7)

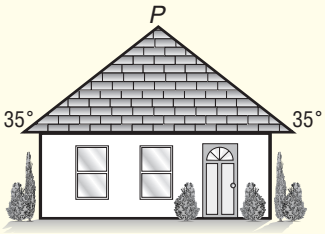
13. Which number is greater, 3.45×10^3 or 5.87×10^2? (Lesson 4-8)

14. Write the value of $\frac{5}{6} - \frac{1}{5} - \frac{1}{30}$ in simplest form. (Lesson 5-4)

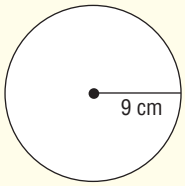
15. Mr. Vazquez budgeted \$300 for his home's January heating bill. The actual bill was \$240. What percent of the \$300 was left after Mr. Vazquez paid the heating bill? (Lesson 6-4)

16. What is the y-intercept of the graph of $y + 5 = 2x$? (Lesson 8-7)

17. Sarah is making a model house. The pitch of the roof is 35°. What is the measure, in degrees, of $\angle P$, the peak of the roof? (Lesson 10-3)

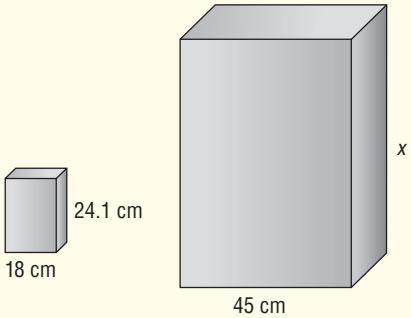


18. What is the circumference of the circle? Use $\pi = 3.14$ and round to the nearest tenth, if necessary. (Lesson 10-9)



19. A concrete worker is making six cement steps. Each step is 4 inches high, 7 inches deep, and 20 inches wide. What volume of cement, in cubic inches, will be needed to make these steps? (Lesson 11-2)

20. The prisms below are similar. Find the height of the larger prism in centimeters. If necessary, round to the nearest tenth. (Lesson 11-6)



Part 3 Extended Response

Record your answers on a sheet of paper. Show your work.

21. A manufacturer ships its product in boxes that are 3 feet × 2 feet × 2 feet. The company needs to store some products in a warehouse space that is 32 feet long by 8 feet wide by 10 feet high. (Lesson 11-2)

a. What is the greatest number of boxes the company can store in this space? (All the boxes must be stored in the same position.)

b. What is the total volume of the stored boxes?

c. What is the volume of the storage space?

d. How much storage space is *not* filled with boxes?