10 Parallel and Perpendicular Lines

10.1 Pairs of Lines and Angles
10.2 Parallel Lines and Transversals
10.3 Proofs with Parallel Lines
10.4 Proofs with Perpendicular Lines
10.5 Using Parallel and Perpendicular Lines

See the Big Idea

Bike Path (p. 532)

Crosswalk (p. 526)

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Tree House (p. 502)
Maintaining Mathematical Proficiency

Finding Angle Measures

Example 1  Find the measure of each angle.

\[
\text{Step 1  Use the fact that the sum of the measures of supplementary angles is } 180^\circ.
\]

\[
m\angle ABD + m\angle DBC = 180^\circ
\]

\[
(8x - 6)^\circ + (9x + 16)^\circ = 180^\circ
\]

\[
17x + 10 = 180
\]

\[
x = 10
\]

\[
\text{Step 2  Evaluate the given expressions when } x = 10.
\]

\[
m\angle ABD = (8x - 6)^\circ = (8 \cdot 10 - 6)^\circ = 74^\circ
\]

\[
m\angle DBC = (9x + 16)^\circ = (9 \cdot 10 + 16)^\circ = 106^\circ
\]

\[
\text{So, } m\angle ABD = 74^\circ \text{ and } m\angle DBC = 106^\circ.
\]

Find the measure of each angle.

1. \[
\text{E}\quad \text{F}\quad \text{G}
\]

2. \[
\text{H}\quad \text{I}\quad \text{J}
\]

3. \[
\text{K}\quad \text{L}\quad \text{M}
\]

Writing Equations of Lines

Example 2  Write an equation of the line that passes through the point \((-4, 5)\) and has a slope of \(\frac{3}{4}\).

\[
y = mx + b
\]

\[
y = \frac{3}{4}(-4) + b
\]

\[
y = -3 + b
\]

\[
y = 8
\]

\[
\text{So, an equation is } y = \frac{3}{4}x + 8.
\]

Write an equation of the line that passes through the given point and has the given slope.

4. \((6, 1); m = -3\)

5. \((-3, 8); m = -2\)

6. \((-1, 5); m = 4\)

7. \((2, -4); m = \frac{1}{2}\)

8. \((-8, -5); m = -\frac{1}{4}\)

9. \((0, 9); m = \frac{2}{3}\)

10. ABSTRACT REASONING  How can you write an equation of a line that passes through the point \((a, b)\) and has an undefined slope?
Characteristics of Lines in a Coordinate Plane

**Core Concept**

**Lines in a Coordinate Plane**
1. In a coordinate plane, two lines are parallel if and only if they are both vertical lines or they both have the same slope.
2. In a coordinate plane, two lines are perpendicular if and only if one is vertical and the other is horizontal or the slopes of the lines are negative reciprocals of each other.
3. In a coordinate plane, two lines are coincident if and only if their equations are equivalent.

**EXAMPLE 1** Classifying Pairs of Lines

Here are some examples of pairs of lines in a coordinate plane.

a. \(2x + y = 2\) and \(x - y = 4\)  
These lines are not parallel or perpendicular. They intersect at \((2, -2)\).

b. \(2x + y = 2\) and \(4x + 2y = 4\)  
These lines are coincident because their equations are equivalent.

c. \(2x + y = 2\) and \(2x + y = 4\)  
These lines are parallel. Each line has a slope of \(m = -2\).

d. \(2x + y = 2\) and \(x - 2y = 4\)  
These lines are perpendicular. They have slopes of \(m_1 = -2\) and \(m_2 = \frac{1}{2}\).

**Monitoring Progress**

Use a graphing calculator to graph the pair of lines. Use a square viewing window. Classify the lines as parallel, perpendicular, coincident, or nonperpendicular intersecting lines. Justify your answer.

1. \(x + 2y = 2\) and \(2x - y = 4\)  
2. \(x + 2y = 2\) and \(2x + 4y = 4\)  
3. \(x + 2y = 2\) and \(x + 2y = -2\)  
4. \(x + 2y = 2\) and \(x - y = -4\)
10.1 Pairs of Lines and Angles

Essential Question What does it mean when two lines are parallel, intersecting, coincident, or skew?

EXPLORATION 1 Points of Intersection

Work with a partner. Write the number of points of intersection of each pair of coplanar lines.

a. parallel lines
b. intersecting lines
c. coincident lines

EXPLORATION 2 Classifying Pairs of Lines

Work with a partner. The figure shows a right rectangular prism. All its angles are right angles. Classify each of the following pairs of lines as parallel, intersecting, coincident, or skew. Justify your answers. (Two lines are skew lines when they do not intersect and are not coplanar.)

Pair of Lines Classification Reason

a. \overline{AB} \text{ and } \overline{BC}

b. \overline{AD} \text{ and } \overline{BC}

c. \overline{EI} \text{ and } \overline{IH}

d. \overline{BF} \text{ and } \overline{EH}

e. \overline{EF} \text{ and } \overline{CG}

f. \overline{AB} \text{ and } \overline{GH}

EXPLORATION 3 Identifying Pairs of Angles

Work with a partner. In the figure, two parallel lines are intersected by a transversal.

a. Identify all the pairs of vertical angles. Explain your reasoning.

b. Identify all the linear pairs of angles. Explain your reasoning.

Communicate Your Answer

4. What does it mean when two lines are parallel, intersecting, coincident, or skew?

5. In Exploration 2, find three more pairs of lines that are different from those given. Classify the pairs of lines as parallel, intersecting, coincident, or skew. Justify your answers.
What You Will Learn

- Identify lines and planes.
- Identify parallel and perpendicular lines.
- Identify pairs of angles formed by transversals.

Identifying Lines and Planes

Core Vocabulary

- skew lines, p. 498
- parallel lines, p. 498
- transversal, p. 500
- corresponding angles, p. 500
- alternate interior angles, p. 500
- alternate exterior angles, p. 500
- consecutive interior angles, p. 500

Core Concept

Parallel Lines, Skew Lines, and Parallel Planes

Two lines that do not intersect are either parallel lines or skew lines. Recall that two lines are parallel lines when they do not intersect and are coplanar. Two lines are skew lines when they do not intersect and are not coplanar. Also, two planes that do not intersect are parallel planes.

- Lines \( m \) and \( n \) are parallel lines \((m \parallel n)\).
- Lines \( m \) and \( k \) are skew lines.
- Planes \( T \) and \( U \) are parallel planes \((T \parallel U)\).
- Lines \( k \) and \( n \) are intersecting lines, and there is a plane (not shown) containing them.

Small directed arrows, as shown in red on lines \( m \) and \( n \) above, are used to show that lines are parallel. The symbol \( \parallel \) means “is parallel to,” as in \( m \parallel n \).

Segments and rays are parallel when they lie in parallel lines. A line is parallel to a plane when the line is in a plane parallel to the given plane. In the diagram above, line \( n \) is parallel to plane \( U \).

Example 1

Identifying Lines and Planes

Think of each segment in the figure as part of a line. Which line(s) or plane(s) appear to fit the description?

a. line(s) parallel to \( CD \) and containing point \( A \)

b. line(s) skew to \( CD \) and containing point \( A \)

c. line(s) perpendicular to \( CD \) and containing point \( A \)

d. plane(s) parallel to plane \( EFG \) and containing point \( A \)

Solution

- a. \( \overline{AB}, \overline{HG}, \) and \( \overline{EF} \) all appear parallel to \( \overline{CD} \), but only \( \overline{AB} \) contains point \( A \).
- b. Both \( \overline{AG} \) and \( \overline{AH} \) appear skew to \( \overline{CD} \) and contain point \( A \).
- c. \( \overline{BC}, \overline{AD}, \overline{DE}, \) and \( \overline{FC} \) all appear perpendicular to \( \overline{CD} \), but only \( \overline{AD} \) contains point \( A \).
- d. Plane \( ABC \) appears parallel to plane \( EFG \) and contains point \( A \).

Monitoring Progress

1. Look at the diagram in Example 1. Name the line(s) through point \( F \) that appear skew to \( \overline{EH} \).
Identifying Parallel and Perpendicular Lines

Two distinct lines in the same plane either are parallel, like line $\ell$ and line $n$, or intersect in a point, like line $j$ and line $n$.

Through a point not on a line, there are infinitely many lines. Exactly one of these lines is parallel to the given line, and exactly one of them is perpendicular to the given line. For example, line $k$ is the line through point $P$ perpendicular to line $\ell$, and line $n$ is the line through point $P$ parallel to line $\ell$.

**Postulates**

**Parallel Postulate**

If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.

There is exactly one line through $P$ parallel to $\ell$.

**Perpendicular Postulate**

If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.

There is exactly one line through $P$ perpendicular to $\ell$.

**EXAMPLE 2** Identifying Parallel and Perpendicular Lines

The given line markings show how the roads in a town are related to one another.

a. Name a pair of parallel lines.

b. Name a pair of perpendicular lines.

c. Is $\overrightarrow{FE} \parallel \overrightarrow{AC}$? Explain.

**SOLUTION**

a. $\overrightarrow{MD} \parallel \overrightarrow{FE}$

b. $\overrightarrow{MD} \perp \overrightarrow{BF}$

c. $\overrightarrow{FE}$ is not parallel to $\overrightarrow{AC}$, because $\overrightarrow{MD}$ is parallel to $\overrightarrow{FE}$, and by the Parallel Postulate, there is exactly one line parallel to $\overrightarrow{FE}$ through $M$.

2. In Example 2, can you use the Perpendicular Postulate to show that $\overrightarrow{AC}$ is not perpendicular to $\overrightarrow{BF}$? Explain why or why not.
Identifying Pairs of Angles

A transversal is a line that intersects two or more coplanar lines at different points.

Core Concept

Angles Formed by Transversals

Two angles are corresponding angles when they have corresponding positions. For example, \( \angle 2 \) and \( \angle 6 \) are above the lines and to the right of the transversal \( t \).

Two angles are alternate interior angles when they lie between the two lines and on opposite sides of the transversal \( t \).

Two angles are alternate exterior angles when they lie outside the two lines and on opposite sides of the transversal \( t \).

Two angles are consecutive interior angles when they lie between the two lines and on the same side of the transversal \( t \).

EXAMPLE 3

Identifying Pairs of Angles

Identify all pairs of angles of the given type.

a. corresponding
b. alternate interior
c. alternate exterior
d. consecutive interior

SOLUTION

a. \( \angle 1 \) and \( \angle 5 \) 
\( \angle 2 \) and \( \angle 6 \) 
\( \angle 3 \) and \( \angle 7 \) 
\( \angle 4 \) and \( \angle 8 \)

b. \( \angle 2 \) and \( \angle 7 \) 
\( \angle 4 \) and \( \angle 5 \)

4. \( \angle 2 \) and \( \angle 5 \) 
\( \angle 3 \) and \( \angle 6 \) 
\( \angle 4 \) and \( \angle 7 \)

Monitoring Progress

Classify the pair of numbered angles.

3. \( \angle 1 \) and \( \angle 5 \) 
4. \( \angle 2 \) and \( \angle 7 \) 
5. \( \angle 5 \) and \( \angle 4 \)
1. **COMPLETE THE SENTENCE** Two lines that do not intersect and are also not parallel are ________ lines.

2. **WHICH ONE DOESN’T BELONG?** Which angle pair does not belong with the other three? Explain your reasoning.
   - ∠2 and ∠3
   - ∠4 and ∠5
   - ∠1 and ∠8
   - ∠2 and ∠7

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### Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, think of each segment in the diagram as part of a line. All the angles are right angles. Which line(s) or plane(s) contain point $B$ and appear to fit the description? *(See Example 1.)*

3. line(s) parallel to $\overrightarrow{CD}$
4. line(s) perpendicular to $\overrightarrow{CD}$
5. line(s) skew to $\overrightarrow{CD}$
6. plane(s) parallel to plane $CDH$

In Exercises 7–10, use the diagram. *(See Example 2.)*

7. Name a pair of parallel lines.
8. Name a pair of perpendicular lines.
9. Is $\overrightarrow{PN} \parallel \overrightarrow{KM}$? Explain.
10. Is $\overrightarrow{PR} \perp \overrightarrow{NP}$? Explain.

In Exercises 11–14, identify all pairs of angles of the given type. *(See Example 3.)*

11. corresponding
12. alternate interior
13. alternate exterior
14. consecutive interior

**USING STRUCTURE** In Exercises 15–18, classify the angle pair as **corresponding**, **alternate interior**, **alternate exterior**, or **consecutive interior angles**.

15. $\angle 5$ and $\angle 1$
16. $\angle 11$ and $\angle 13$
17. $\angle 6$ and $\angle 13$
18. $\angle 2$ and $\angle 11$
ERROR ANALYSIS  In Exercises 19 and 20, describe and correct the error in the conditional statement about lines.

19. \(\text{If two lines do not intersect, then they are parallel.}\)

20. \(\text{If there is a line and a point not on the line, then there is exactly one line through the point that intersects the given line.}\)

21. MODELING WITH MATHEMATICS Use the photo to decide whether the statement is true or false. Explain your reasoning.

a. The plane containing the floor of the tree house is parallel to the ground.

b. The lines containing the railings of the staircase, such as \(\overline{AB}\), are skew to all lines in the plane containing the ground.

c. All the lines containing the balusters, such as \(\overline{CD}\), are perpendicular to the plane containing the floor of the tree house.

22. THOUGHT PROVOKING If two lines are intersected by a third line, is the third line necessarily a transversal? Justify your answer with a diagram.

23. MATHEMATICAL CONNECTIONS Two lines are cut by a transversal. Is it possible for all eight angles formed to have the same measure? Explain your reasoning.

24. HOW DO YOU SEE IT? Think of each segment in the figure as part of a line.

   a. Which lines are parallel to \(\overline{NQ}\)?
   
   b. Which lines intersect \(\overline{NQ}\)?
   
   c. Which lines are skew to \(\overline{NQ}\)?
   
   d. Should you have named all the lines on the cube in parts (a)–(c) except \(\overline{NQ}\)? Explain.

In Exercises 25–28, copy and complete the statement. List all possible correct answers.

25. \(\angle BCG\) and ____ are corresponding angles.

26. \(\angle BCG\) and ____ are consecutive interior angles.

27. \(\angle FCJ\) and ____ are alternate interior angles.

28. \(\angle FCA\) and ____ are alternate exterior angles.

29. MAKING AN ARGUMENT Your friend claims the uneven parallel bars in gymnastics are not really parallel. She says one is higher than the other, so they cannot be in the same plane. Is she correct? Explain.

Maintaining Mathematical Proficiency

Use the diagram to find the measures of all the angles. (Section 9.5)

30. \(m\angle 1 = 76^\circ\)

31. \(m\angle 2 = 159^\circ\)
10.2 Parallel Lines and Transversals

**Essential Question** When two parallel lines are cut by a transversal, which of the resulting pairs of angles are congruent?

**Exploration 1** Exploring Parallel Lines

Work with a partner.
Use dynamic geometry software to draw two parallel lines. Draw a third line that intersects both parallel lines. Find the measures of the eight angles that are formed. What can you conclude?

**Exploration 2** Writing Conjectures

Work with a partner. Use the results of Exploration 1 to write conjectures about the following pairs of angles formed by two parallel lines and a transversal.

- a. corresponding angles
- b. alternate interior angles
- c. alternate exterior angles
- d. consecutive interior angles

**Communicate Your Answer**

3. When two parallel lines are cut by a transversal, which of the resulting pairs of angles are congruent?
4. In Exploration 2, \( m\angle 1 = 80^\circ \). Find the other angle measures.
What You Will Learn

- Use properties of parallel lines.
- Prove theorems about parallel lines.
- Solve real-life problems.

Using Properties of Parallel Lines

### Core Vocabulary

- Corresponding angles
- Parallel lines
- Supplementary angles
- Vertical angles

### Theorems

#### Corresponding Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

**Examples**

In the diagram at the left, \( \angle 2 \cong \angle 6 \) and \( \angle 3 \cong \angle 7 \).

**Proof**

Ex. 36, p. 550

#### Alternate Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

**Examples**

In the diagram at the left, \( \angle 3 \cong \angle 6 \) and \( \angle 4 \cong \angle 5 \).

**Proof**

Example 4, p. 506

#### Alternate Exterior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

**Examples**

In the diagram at the left, \( \angle 1 \cong \angle 8 \) and \( \angle 2 \cong \angle 7 \).

**Proof**

Ex. 15, p. 508

#### Consecutive Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.

**Examples**

In the diagram at the left, \( \angle 3 \) and \( \angle 5 \) are supplementary, and \( \angle 4 \) and \( \angle 6 \) are supplementary.

**Proof**

Ex. 16, p. 508

### Example 1: Identifying Angles

The measures of three of the numbered angles are 120°. Identify the angles. Explain your reasoning.

**Solution**

By the Alternate Exterior Angles Theorem, \( \angle 8 = 120^\circ \).

\( \angle 5 \) and \( \angle 8 \) are vertical angles. Using the Vertical Angles Congruence Theorem, \( \angle 5 = 120^\circ \).

\( \angle 5 \) and \( \angle 4 \) are alternate interior angles. By the Alternate Interior Angles Theorem, \( \angle 4 = 120^\circ \).

So, the three angles that each have a measure of 120° are \( \angle 4 \), \( \angle 5 \), and \( \angle 8 \).
**EXAMPLE 2**  Using Properties of Parallel Lines

Find the value of $x$.

\[
\begin{align*}
\angle 4 + (x + 5)^\circ &= 180^\circ \\
115^\circ + (x + 5)^\circ &= 180^\circ \\
x + 120 &= 180 \\
x &= 60
\end{align*}
\]

So, the value of $x$ is 60.

**EXAMPLE 3**  Using Properties of Parallel Lines

Find the value of $x$.

\[
\begin{align*}
\angle 1 &= (7x + 9)^\circ \\
44^\circ &= (7x + 9)^\circ \\
35 &= 7x \\
5 &= x
\end{align*}
\]

So, the value of $x$ is 5.

**Monitoring Progress**

Use the diagram.

1. Given $\angle 1 = 105^\circ$, find $\angle 4$, $\angle 5$, and $\angle 8$. Tell which theorem you use in each case.

2. Given $\angle 3 = 68^\circ$ and $\angle 8 = (2x + 4)^\circ$, what is the value of $x$? Show your steps.
**Proving Theorems about Parallel Lines**

**EXAMPLE 4** Proving the Alternate Interior Angles Theorem

Prove that if two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

**SOLUTION**

Draw a diagram. Label a pair of alternate interior angles as $\angle 1$ and $\angle 2$. You are looking for an angle that is related to both $\angle 1$ and $\angle 2$. Notice that one angle is a vertical angle with $\angle 2$ and a corresponding angle with $\angle 1$. Label it $\angle 3$.

**Given** $p \parallel q$

**Prove** $\angle 1 \cong \angle 2$

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $p \parallel q$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle 1 \cong \angle 3$</td>
<td>2. Corresponding Angles Theorem</td>
</tr>
<tr>
<td>3. $\angle 3 \cong \angle 2$</td>
<td>3. Vertical Angles Congruence Theorem</td>
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<td>4. $\angle 1 \cong \angle 2$</td>
<td>4. Transitive Property of Congruence</td>
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3. In the proof in Example 4, if you use the third statement before the second statement, could you still prove the theorem? Explain.

**Solving Real-Life Problems**

**EXAMPLE 5** Solving a Real-life Problem

When sunlight enters a drop of rain, different colors of light leave the drop at different angles. This process is what makes a rainbow. For violet light, $m\angle 2 = 40^\circ$. What is $m\angle 1$? How do you know?

**SOLUTION**

Because the Sun’s rays are parallel, $\angle 1$ and $\angle 2$ are alternate interior angles. By the Alternate Interior Angles Theorem, $\angle 1 \cong \angle 2$.

So, by the definition of congruent angles, $m\angle 1 = m\angle 2 = 40^\circ$.

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4. **WHAT IF?** In Example 5, yellow light leaves a drop at an angle of $m\angle 2 = 41^\circ$. What is $m\angle 1$? How do you know?
10.2 Exercises

Vocabulary and Core Concept Check

1. WRITING How are the Alternate Interior Angles Theorem and the Alternate Exterior Angles Theorem alike? How are they different?

2. WHICH ONE DOESN’T BELONG? Which pair of angle measures does not belong with the other three? Explain.

   - $m\angle 1$ and $m\angle 3$
   - $m\angle 2$ and $m\angle 4$
   - $m\angle 2$ and $m\angle 3$
   - $m\angle 1$ and $m\angle 5$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find $m\angle 1$ and $m\angle 2$. Tell which theorem you use in each case. (See Example 1.)

3. $\angle 1 = 117^\circ$
4. $\angle 1 = 150^\circ$
5. $\angle 1 = 122^\circ$
6. $\angle 1 = 140^\circ$

In Exercises 7–10, find the value of $x$. Show your steps. (See Examples 2 and 3.)

7. $2x^\circ = 128^\circ$
8. $72^\circ = (7x + 24)^\circ$
9. $65^\circ = 5(11x - 17)^\circ$

In Exercises 11 and 12, find $m\angle 1$, $m\angle 2$, and $m\angle 3$. Explain your reasoning.

11. $\angle 1 = 80^\circ$, $\angle 2 = 3^\circ$
12. $\angle 1 = 133^\circ$

13. ERROR ANALYSIS Describe and correct the error in the student’s reasoning.

\[ \angle 9 \cong \angle 10 \text{ by the Corresponding Angles Theorem.} \]
14. **HOW DO YOU SEE IT?**

Use the diagram.

- a. Name two pairs of congruent angles when $\overline{AD}$ and $\overline{BC}$ are parallel. Explain your reasoning.
- b. Name two pairs of supplementary angles when $\overline{AB}$ and $\overline{DC}$ are parallel. Explain your reasoning.

**PROVING A THEOREM** In Exercises 15 and 16, prove the theorem. (See Example 4.)

15. Alternate Exterior Angles Theorem

16. Consecutive Interior Angles Theorem

**PROBLEM SOLVING**

A group of campers tie up their food between two parallel trees, as shown. The rope is pulled taut, forming a straight line. Find $m\angle 2$. Explain your reasoning. (See Example 5.)

17. **DRAWING CONCLUSIONS** You are designing a box like the one shown.

- a. The measure of $\angle 1$ is $70^\circ$. Find $m\angle 2$ and $m\angle 3$.
- b. Explain why $\angle ABC$ is a straight angle.
- c. If $m\angle 1$ is $60^\circ$, will $\angle ABC$ still be a straight angle? Will the opening of the box be more steep or less steep? Explain.

18. **MAKING AN ARGUMENT** During a game of pool, your friend claims to be able to make the shot shown in the diagram by hitting the cue ball so that $m\angle 1 = 25^\circ$. Is your friend correct? Explain your reasoning.

19. **CRITICAL THINKING** Is it possible for consecutive interior angles to be congruent? Explain.

20. **THOUGHT PROVOKING** The postulates and theorems in this book represent Euclidean geometry. In spherical geometry, all points are points on the surface of a sphere. A line is a circle on the sphere whose diameter is equal to the diameter of the sphere. In spherical geometry, is it possible that a transversal intersects two parallel lines? Explain your reasoning.

**MATHEMATICAL CONNECTIONS** In Exercises 21 and 22, write and solve a system of linear equations to find the values of $x$ and $y$.

21. $2y - (14x - 10)^\circ$

22. $2y^\circ = 4x^\circ$

23. $5x^\circ = (2x + 12)^\circ (y + 6)^\circ$

24. **REASONING** In the diagram, $\angle 4 \equiv \angle 5$ and $\overline{SE}$ bisects $\angle RSF$. Find $m\angle 1$. Explain your reasoning.

**Maintaining Mathematical Proficiency**

Write the converse of the conditional statement. Decide whether it is true or false. (Section 9.1)

25. If two angles are vertical angles, then they are congruent.

26. If you go to the zoo, then you will see a tiger.

27. If two angles form a linear pair, then they are supplementary.

28. If it is warm outside, then we will go to the park.
**Essential Question:** For which of the theorems involving parallel lines and transversals is the converse true?

**EXPLORATION 1** Exploring Converses

**Work with a partner:** Write the converse of each conditional statement. Draw a diagram to represent the converse. Determine whether the converse is true. Justify your conclusion.

**a. Corresponding Angles Theorem**  If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

**Converse**

**b. Alternate Interior Angles Theorem**  If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

**Converse**

**c. Alternate Exterior Angles Theorem**  If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

**Converse**

**d. Consecutive Interior Angles Theorem**  If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.

**Converse**

**Communicate Your Answer**

2. For which of the theorems involving parallel lines and transversals is the converse true?

3. In Exploration 1, explain how you would prove any of the theorems that you found to be true.
What You Will Learn

- Use the Corresponding Angles Converse.
- Construct parallel lines.
- Prove theorems about parallel lines.
- Use the Transitive Property of Parallel Lines.

Using the Corresponding Angles Converse

The theorem below is the converse of the Corresponding Angles Theorem. Similarly, the other theorems about angles formed when parallel lines are cut by a transversal have true converses. Remember that the converse of a true conditional statement is not necessarily true, so you must prove each converse of a theorem.

**Corresponding Angles Converse**

If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel.

**Proof** Ex. 36, p. 550

**EXAMPLE 1** Using the Corresponding Angles Converse

Find the value of $x$ that makes $m \parallel n$.

**SOLUTION**

Lines $m$ and $n$ are parallel when the marked corresponding angles are congruent.

$(3x + 5)° = 65°$  
Use the Corresponding Angles Converse to write an equation.

$3x = 60$  
Subtract 5 from each side.

$x = 20$  
Divide each side by 3.

So, lines $m$ and $n$ are parallel when $x = 20$.

Monitoring Progress

1. Is there enough information in the diagram to conclude that $m \parallel n$? Explain.

2. Explain why the Corresponding Angles Converse is the converse of the Corresponding Angles Theorem.
Constructing Parallel Lines

The Corresponding Angles Converse justifies the construction of parallel lines, as shown below.

**CONSTRUCTION**

Constructing Parallel Lines

Use a compass and straightedge to construct a line through point \( P \) that is parallel to line \( m \).

**SOLUTION**

**Step 1**

![Diagram](image1)

Draw a point and line
Start by drawing point \( P \) and line \( m \). Choose a point \( Q \) anywhere on line \( m \) and draw \( \overline{QP} \).

**Step 2**

![Diagram](image2)

Draw arcs
Draw an arc with center \( Q \) that crosses \( \overline{QP} \) and line \( m \). Label points \( A \) and \( B \). Using the same compass setting, draw an arc with center \( P \). Label point \( C \).

**Step 3**

![Diagram](image3)

Copy angle
Draw an arc with radius \( AB \) and center \( A \). Using the same compass setting, draw an arc with center \( C \). Label the intersection \( D \).

**Step 4**

![Diagram](image4)

Draw parallel lines
Draw \( \overline{PD} \). This line is parallel to line \( m \).

**Theorems**

**Alternate Interior Angles Converse**

If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

*Proof* Example 2, p. 512

**Alternate Exterior Angles Converse**

If two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are parallel.

*Proof* Ex. 11, p. 514

**Consecutive Interior Angles Converse**

If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are parallel.

*Proof* Ex. 12, p. 514

If \( \angle 3 \) and \( \angle 5 \) are supplementary, then \( j \parallel k \).
Proving Theorems about Parallel Lines

**EXAMPLE 2** Proving the Alternate Interior Angles Converse

Prove that if two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

**SOLUTION**

Given \( \angle 4 \cong \angle 5 \)

Prove \( g \parallel h \)

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
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<tbody>
<tr>
<td>1. ( \angle 4 \cong \angle 5 )</td>
<td>1. Given</td>
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<tr>
<td>2. ( \angle 1 \cong \angle 4 )</td>
<td>2. Vertical Angles Congruence Theorem</td>
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<td>3. ( \angle 1 \cong \angle 5 )</td>
<td>3. Transitive Property of Congruence</td>
</tr>
<tr>
<td>4. ( g \parallel h )</td>
<td>4. Corresponding Angles Converse</td>
</tr>
</tbody>
</table>

**EXAMPLE 3** Determining Whether Lines Are Parallel

In the diagram, \( r \parallel s \) and \( \angle 1 \) is congruent to \( \angle 3 \). Prove \( p \parallel q \).

**SOLUTION**

Look at the diagram to make a plan. The diagram suggests that you look at angles 1, 2, and 3. Also, you may find it helpful to focus on one pair of lines and one transversal at a time.

**Plan for Proof**

a. Look at \( \angle 1 \) and \( \angle 2 \). \( \angle 1 \cong \angle 2 \) because \( r \parallel s \).

b. Look at \( \angle 2 \) and \( \angle 3 \). If \( \angle 2 \cong \angle 3 \), then \( p \parallel q \).

**Plan for Action**

a. It is given that \( r \parallel s \), so by the Corresponding Angles Theorem, \( \angle 1 \cong \angle 2 \).

b. It is also given that \( \angle 1 \cong \angle 3 \). Then \( \angle 2 \cong \angle 3 \) by the Transitive Property of Congruence.

So, by the Alternate Interior Angles Converse, \( p \parallel q \).

**Monitoring Progress**

3. If you use the diagram below to prove the Alternate Exterior Angles Converse, what **Given** and **Prove** statements would you use?

4. Copy and complete the following paragraph proof of the Alternate Interior Angles Converse using the diagram in Example 2.

It is given that \( \angle 4 \cong \angle 5 \). By the _____, \( \angle 1 \cong \angle 4 \). Then by the Transitive Property of Congruence, _____ So, by the _____, \( g \parallel h \).
Using the Transitive Property of Parallel Lines

**Theorem**

**Transitive Property of Parallel Lines**

If two lines are parallel to the same line, then they are parallel to each other.

*Proof* Ex. 39, p. 516; Ex. 25, p. 532

The flag of the United States has 13 alternating red and white stripes. Each stripe is parallel to the stripe immediately below it. Explain why the top stripe is parallel to the bottom stripe.

**Solution**

You can name the stripes from top to bottom as $s_1, s_2, s_3, \ldots , s_{13}$. Each stripe is parallel to the one immediately below it, so $s_1 \parallel s_2, s_2 \parallel s_3$, and so on. Then $s_1 \parallel s_3$ by the Transitive Property of Parallel Lines. Similarly, because $s_3 \parallel s_4$, it follows that $s_1 \parallel s_4$. By continuing this reasoning, $s_1 \parallel s_{13}$.

So, the top stripe is parallel to the bottom stripe.

**5.** Each step is parallel to the step immediately above it. The bottom step is parallel to the ground. Explain why the top step is parallel to the ground.

**6.** In the diagram below, $p \parallel q$ and $q \parallel r$. Find $m \angle 8$. Explain your reasoning.
10.3 Exercises

Vocabulary and Core Concept Check

1. **VOCABULARY** Two lines are cut by a transversal. Which angle pairs must be congruent for the lines to be parallel?

2. **WRITING** Use the theorems from Section 10.2 and the converses of those theorems in this section to write three biconditional statements about parallel lines and transversals.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, find the value of \(x\) that makes \(m \parallel n\). Explain your reasoning. *(See Example 1.)*

3. \[ \begin{align*} \text{m} & \quad \text{n} \\ 120^\circ & \quad 3x^\circ \end{align*} \]

4. \[ \begin{align*} \text{m} & \quad \text{n} \\ (2x + 15)^\circ & \quad 135^\circ \end{align*} \]

5. \[ \begin{align*} \text{m} & \quad \text{n} \\ 150^\circ & \quad (3x - 15)^\circ \end{align*} \]

6. \[ \begin{align*} \text{m} & \quad \text{n} \\ (180 - x)^\circ & \quad x^\circ \end{align*} \]

7. \[ \begin{align*} \text{m} & \quad \text{n} \\ x^\circ & \quad 2x^\circ \end{align*} \]

8. \[ \begin{align*} \text{m} & \quad \text{n} \\ (2x + 20)^\circ & \quad 3x^\circ \end{align*} \]

In Exercises 9 and 10, use a compass and straightedge to construct a line through point \(P\) that is parallel to line \(m\).

9. \(P\) \hspace{1cm} \(m\)

10. \(P\) \hspace{1cm} \(m\)

In Exercises 13–18, decide whether there is enough information to prove that \(m \parallel n\). If so, state the theorem you would use. *(See Example 3.)*

13. \[ \begin{align*} \text{m} & \quad \text{n} \\ r & \quad s \end{align*} \]

14. \[ \begin{align*} \text{m} & \quad \text{n} \\ r & \quad s \end{align*} \]

15. \[ \begin{align*} \text{m} & \quad \text{n} \\ r & \quad s \end{align*} \]

16. \[ \begin{align*} \text{m} & \quad \text{n} \\ r & \quad s \end{align*} \]

17. \[ \begin{align*} \text{m} & \quad \text{n} \\ r & \quad s \end{align*} \]

18. \[ \begin{align*} \text{m} & \quad \text{n} \\ r & \quad s \end{align*} \]

**ERROR ANALYSIS** In Exercises 19 and 20, describe and correct the error in the reasoning.

19. \[ \begin{align*} a & \quad b \quad c \\ x^\circ & \quad y^\circ & \quad z^\circ \end{align*} \]

Conclusion: \(a \parallel b\)

20. \[ \begin{align*} a & \quad b \quad c \\ 1 & \quad 2 & \quad 3 \end{align*} \]

Conclusion: \(a \parallel b\)

PROVING A THEOREM In Exercises 11 and 12, prove the theorem. *(See Example 2.)*

11. Alternate Exterior Angles Converse

12. Consecutive Interior Angles Converse
In Exercises 21–24, are $\overline{AC}$ and $\overline{DF}$ parallel? Explain your reasoning.

21.  

22.  

23.  

24.  

25. **ANALYZING RELATIONSHIPS** The map shows part of Denver, Colorado. Use the markings on the map. Are the numbered streets parallel to one another? Explain your reasoning. (See Example 4.)

26. **ANALYZING RELATIONSHIPS** Each rung of the ladder is parallel to the rung directly above it. Explain why the top rung is parallel to the bottom rung.

27. **MODELING WITH MATHEMATICS** The diagram of the control bar of the kite shows the angles formed between the control bar and the kite lines. How do you know that $n$ is parallel to $m$?

28. **REASONING** Use the diagram. Which rays are parallel? Which rays are not parallel? Explain your reasoning.

29. **ATTENDING TO PRECISION** Use the diagram. Which theorems allow you to conclude that $m \parallel n$? Select all that apply. Explain your reasoning.

30. **MODELING WITH MATHEMATICS** One way to build stairs is to attach triangular blocks to an angled support, as shown. The sides of the angled support are parallel. If the support makes a $32^\circ$ angle with the floor, what must $m \angle 1$ be so the top of the step will be parallel to the floor? Explain your reasoning.

31. **ABSTRACT REASONING** In the diagram, how many angles must be given to determine whether $j \parallel k$? Give four examples that would allow you to conclude that $j \parallel k$ using the theorems from this lesson.
32. **THOUGHT PROVOKING** Draw a diagram of at least two lines cut by at least one transversal. Mark your diagram so that it cannot be proven that any lines are parallel. Then explain how your diagram would need to change in order to prove that lines are parallel.

**PROOF** In Exercises 33–36, write a proof.

33. **Given** \( m \angle 1 = 115^\circ, m \angle 2 = 65^\circ \)
   **Prove** \( m \parallel n \)

34. **Given** \( \angle 1 \) and \( \angle 3 \) are supplementary.
   **Prove** \( m \parallel n \)

35. **Given** \( \angle 1 \equiv \angle 2, \angle 3 \equiv \angle 4 \)
   **Prove** \( AB \parallel CD \)

36. **Given** \( a \parallel b, \angle 2 \equiv \angle 3 \)
   **Prove** \( c \parallel d \)

37. **MAKING AN ARGUMENT** Your classmate decided that \( AD \parallel BC \) based on the diagram. Is your classmate correct? Explain your reasoning.

38. **HOW DO YOU SEE IT?** Are the markings on the diagram enough to conclude that any lines are parallel? If so, which ones? If not, what other information is needed?

39. **PROVING A THEOREM** Use these steps to prove the Transitive Property of Parallel Lines Theorem.
   a. Copy the diagram with the Transitive Property of Parallel Lines Theorem on page 513.
   b. Write the **Given** and **Prove** statements.
   c. Use the properties of angles formed by parallel lines cut by a transversal to prove the theorem.

40. **MATHEMATICAL CONNECTIONS** Use the diagram.
   a. Find the value of \( x \) that makes \( p \parallel q \).
   b. Find the value of \( y \) that makes \( r \parallel s \).
   c. Can \( r \) be parallel to \( s \) and can \( p \) be parallel to \( q \) at the same time? Explain your reasoning.

**Maintaining Mathematical Proficiency** Reviewing what you learned in previous grades and lessons

**Use the Distance Formula to find the distance between the two points.** *(Section 8.3)*

41. \((1, 3)\) and \((-2, 9)\)  
42. \((-3, 7)\) and \((8, -6)\)  
43. \((5, -4)\) and \((0, 8)\)  
44. \((13, 1)\) and \((9, -4)\)
10.1–10.3 What Did You Learn?

Core Vocabulary

skew lines, p. 498
parallel planes, p. 498
transversal, p. 500
consecutive angles, p. 500
alternate interior angles, p. 500
alternate exterior angles, p. 500
consecutive interior angles, p. 500

Core Concepts

Section 10.1
Parallel Lines, Skew Lines, and Parallel Planes, p. 498
Parallel Postulate, p. 499
Perpendicular Postulate, p. 499
Angles Formed by Transversals, p. 500

Section 10.2
Corresponding Angles Theorem, p. 504
Alternate Interior Angles Theorem, p. 504
Alternate Exterior Angles Theorem, p. 504
Consecutive Interior Angles Theorem, p. 504

Section 10.3
Corresponding Angles Converse, p. 510
Alternate Interior Angles Converse, p. 511
Alternate Exterior Angles Converse, p. 511
Consecutive Interior Angles Converse, p. 511
Transitive Property of Parallel Lines, p. 513

Mathematical Practices

1. Draw the portion of the diagram that you used to answer Exercise 26 on page 502.
2. In Exercise 40 on page 516, explain how you started solving the problem and why you started that way.

Visual Learners

Draw a picture of a word problem.
• Draw a picture of a word problem before starting to solve the problem. You do not have to be an artist.
• When making a review card for a word problem, include a picture. This will help you recall the information while taking a test.
• Make sure your notes are visually neat for easy recall.
10.1–10.3 Quiz

Think of each segment in the diagram as part of a line. Which line(s) or plane(s) contain point $G$ and appear to fit the description? (Section 10.1)

1. line(s) parallel to $\overrightarrow{EF}$
2. line(s) perpendicular to $\overrightarrow{EF}$
3. line(s) skew to $\overrightarrow{EF}$
4. plane(s) parallel to plane $ADE$

Identify all pairs of angles of the given type. (Section 10.1)

5. consecutive interior
6. alternate interior
7. corresponding
8. alternate exterior

Find $m\angle 1$ and $m\angle 2$. Tell which theorem you use in each case. (Section 10.2)

9. $\angle 1 = 138°$
10. $\angle 2 = 123°$
11. $\angle 1 = 57°$

Decide whether there is enough information to prove that $m \parallel n$. If so, state the theorem you would use. (Section 10.3)

12. $m = 69°$, $n = 111°$
13. $m \perp n$
14. $\ell \parallel m$ and $\ell \parallel n$

15. Cellular phones use bars like the ones shown to indicate how much signal strength a phone receives from the nearest service tower. Each bar is parallel to the bar directly next to it. (Section 10.3)
   a. Explain why the tallest bar is parallel to the shortest bar.
   b. Imagine that the left side of each bar extends infinitely as a line. If $m\angle 1 = 58°$, then what is $m\angle 2$?

16. The diagram shows lines formed on a tennis court. (Section 10.1 and Section 10.3)
   a. Identify two pairs of parallel lines so that each pair is in a different plane.
   b. Identify two pairs of perpendicular lines.
   c. Identify two pairs of skew lines.
   d. Prove that $\angle 1 \cong \angle 2$. 

518 Chapter 10 Parallel and Perpendicular Lines
Essential Question: What conjectures can you make about perpendicular lines?

EXPLORATION 1 Writing Conjectures
Work with a partner. Fold a piece of paper in half twice. Label points on the two creases, as shown.

a. Write a conjecture about \(\overline{AB}\) and \(\overline{CD}\). Justify your conjecture.

b. Write a conjecture about \(\overline{AO}\) and \(\overline{OB}\). Justify your conjecture.

EXPLORATION 2 Exploring a Segment Bisector
Work with a partner. Fold and crease a piece of paper, as shown. Label the ends of the crease as \(A\) and \(B\).

a. Fold the paper again so that point \(A\) coincides with point \(B\). Crease the paper on that fold.

b. Unfold the paper and examine the four angles formed by the two creases. What can you conclude about the four angles?

EXPLORATION 3 Writing a Conjecture
Work with a partner.

a. Draw \(\overline{AB}\), as shown.

b. Draw an arc with center \(A\) on each side of \(\overline{AB}\). Using the same compass setting, draw an arc with center \(B\) on each side of \(\overline{AB}\). Label the intersections of the arcs \(C\) and \(D\).

c. Draw \(\overline{CD}\). Label its intersection with \(\overline{AB}\) as \(O\). Write a conjecture about the resulting diagram. Justify your conjecture.

Communicate Your Answer

4. What conjectures can you make about perpendicular lines?

5. In Exploration 3, find \(AO\) and \(OB\) when \(AB = 4\) units.
What You Will Learn

- Find the distance from a point to a line.
- Construct perpendicular lines.
- Prove theorems about perpendicular lines.
- Solve real-life problems involving perpendicular lines.

Finding the Distance from a Point to a Line

The **distance from a point to a line** is the length of the perpendicular segment from the point to the line. This perpendicular segment is the shortest distance between the point and the line. For example, the distance between point \( A \) and line \( k \) is \( AB \).

**Example 1** Finding the Distance from a Point to a Line

Find the distance from point \( A \) to \( \overrightarrow{BD} \).

**Solution**

Because \( AC \perp \overrightarrow{BD} \), the distance from point \( A \) to \( \overrightarrow{BD} \) is \( AC \). Use the Distance Formula.

\[
AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

So, the distance from point \( A \) to \( \overrightarrow{BD} \) is about 5.7 units.

**Monitoring Progress**

1. Find the distance from point \( E \) to \( \overrightarrow{FH} \).
Constructing Perpendicular Lines

**CONSTRUCTION**

**Constructing a Perpendicular Line**

Use a compass and straightedge to construct a line perpendicular to line \( m \) through point \( P \), which is not on line \( m \).

**SOLUTION**

**Step 1**

Draw arc with center \( P \) Place the compass at point \( P \) and draw an arc that intersects the line twice. Label the intersections \( A \) and \( B \).

**Step 2**

Draw intersecting arcs Draw an arc with center \( A \). Using the same radius, draw an arc with center \( B \). Label the intersection of the arcs \( Q \).

**Step 3**

Draw perpendicular line Draw \( \overline{PQ} \). This line is perpendicular to line \( m \).

The **perpendicular bisector** of a line segment \( \overline{PQ} \) is the line \( n \) with the following two properties.

- \( n \perp \overline{PQ} \)
- \( n \) passes through the midpoint \( M \) of \( \overline{PQ} \).

**CONSTRUCTION**

**Constructing a Perpendicular Bisector**

Use a compass and straightedge to construct the perpendicular bisector of \( \overline{AB} \).

**SOLUTION**

**Step 1**

Draw an arc Place the compass at \( A \). Use a compass setting that is greater than half the length of \( \overline{AB} \). Draw an arc.

**Step 2**

Draw a second arc Keep the same compass setting. Place the compass at \( B \). Draw an arc. It should intersect the other arc at two points.

**Step 3**

Bisect segment Draw a line through the two points of intersection. This line is the perpendicular bisector of \( \overline{AB} \). It passes through \( M \), the midpoint of \( \overline{AB} \). So, \( AM = MB \).
Proving Theorems about Perpendicular Lines

**Linear Pair Perpendicular Theorem**
If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.
If \( \angle 1 \cong \angle 2 \), then \( g \perp h \).
*Proof* Ex. 13, p. 525

**Perpendicular Transversal Theorem**
In a plane, if a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other line.
If \( h \parallel k \) and \( j \perp h \), then \( j \perp k \).
*Proof* Example 2, p. 522; Question 2, p. 522

**Lines Perpendicular to a Transversal Theorem**
In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.
If \( m \perp p \) and \( n \perp p \), then \( m \parallel n \).
*Proof* Ex. 14, p. 525; Ex. 24, p. 532

---

**EXAMPLE 2** Proving the Perpendicular Transversal Theorem

Use the diagram to prove the Perpendicular Transversal Theorem.

**SOLUTION**

**Given** \( h \parallel k, j \perp h \)

**Prove** \( j \perp k \)

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<td>1. ( h \parallel k, j \perp h )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( m\angle 2 = 90^\circ )</td>
<td>2. Definition of perpendicular lines</td>
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<tr>
<td>3. ( \angle 2 \cong \angle 6 )</td>
<td>3. Corresponding Angles Theorem</td>
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<tr>
<td>4. ( m\angle 2 = m\angle 6 )</td>
<td>4. Definition of congruent angles</td>
</tr>
<tr>
<td>5. ( m\angle 6 = 90^\circ )</td>
<td>5. Transitive Property of Equality</td>
</tr>
<tr>
<td>6. ( j \perp k )</td>
<td>6. Definition of perpendicular lines</td>
</tr>
</tbody>
</table>

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**Monitoring Progress**

2. Prove the Perpendicular Transversal Theorem using the diagram in Example 2 and the Alternate Exterior Angles Theorem.
Solving Real-Life Problems

EXAMPLE 3 Proving Lines Are Parallel

The photo shows the layout of a neighborhood. Determine which lines, if any, must be parallel in the diagram. Explain your reasoning.

SOLUTION

Lines \( p \) and \( q \) are both perpendicular to \( s \), so by the Lines Perpendicular to a Transversal Theorem, \( p \parallel q \). Also, lines \( s \) and \( t \) are both perpendicular to \( q \), so by the Lines Perpendicular to a Transversal Theorem, \( s \parallel t \).

So, from the diagram you can conclude \( p \parallel q \) and \( s \parallel t \).

Monitoring Progress

Use the lines marked in the photo.

3. Is \( b \parallel a \)? Explain your reasoning.

4. Is \( b \perp c \)? Explain your reasoning.
1. **COMPLETE THE SENTENCE** The perpendicular bisector of a segment is the line that passes through the ________ of the segment at a ________ angle.

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find “both” answers.

   Find the distance from point $X$ to line $WZ$.
   Find $XZ$.
   Find the length of $XY$.
   Find the distance from line $\ell$ to point $X$.

---

**Monitoring Progress and Modeling with Mathematics**

In Exercises 3 and 4, find the distance from point $A$ to $XZ$. (See Example 1.)

3. ![Graph](image1.jpg)

4. ![Graph](image2.jpg)

---

**CONSTRUCTION** In Exercises 5–8, trace line $m$ and point $P$. Then use a compass and straightedge to construct a line perpendicular to line $m$ through point $P$.

5. ![Graph](image3.jpg)

6. ![Graph](image4.jpg)

7. ![Graph](image5.jpg)

8. ![Graph](image6.jpg)

**CONSTRUCTION** In Exercises 9 and 10, trace $AB$. Then use a compass and straightedge to construct the perpendicular bisector of $AB$.

9. ![Graph](image7.jpg)

10. ![Graph](image8.jpg)
**ERROR ANALYSIS** In Exercises 11 and 12, describe and correct the error in the statement about the diagram.

11. Lines $y$ and $z$ are parallel.

12. The distance from point $C$ to $\overrightarrow{AB}$ is 12 centimeters.

**PROVING A THEOREM** In Exercises 13 and 14, prove the theorem. (See Example 2.)

13. Linear Pair Perpendicular Theorem

14. Lines Perpendicular to a Transversal Theorem

**PROOF** In Exercises 15 and 16, use the diagram to write a proof of the statement.

15. If two intersecting lines are perpendicular, then they intersect to form four right angles.

   **Given** $a \perp b$
   **Prove** $\angle 1, \angle 2, \angle 3,$ and $\angle 4$ are right angles.

16. If two sides of two adjacent acute angles are perpendicular, then the angles are complementary.

   **Given** $\overline{BA} \perp \overline{BC}$
   **Prove** $\angle 1$ and $\angle 2$ are complementary.

**In Exercises 17–22, determine which lines, if any, must be parallel. Explain your reasoning. (See Example 3.)**

17.

18.

19.

20.

21.

22.

**USING STRUCTURE** Find all the unknown angle measures in the diagram. Justify your answer for each angle measure.

23.

**MAKING AN ARGUMENT** Your friend claims that because you can find the distance from a point to a line, you should be able to find the distance between any two lines. Is your friend correct? Explain your reasoning.

24.

25. **MATHEMATICAL CONNECTIONS** Find the value of $x$ when $a \perp b$ and $b \parallel c$.

   $\triangle 1$ $\triangle 2$ $\triangle 3$ $\triangle 4$

   $9x + 18$ 1

   $5(x + 7) + 15$

   $c$

   $b$

   $a$
26. **HOW DO YOU SEE IT?** You are trying to cross a stream from point A. Which point should you jump to in order to jump the shortest distance? Explain your reasoning.

27. **ATTENDING TO PRECISION** In which of the following diagrams is AC \parallel BD and AC \perp CD? Select all that apply.

28. **THOUGHT PROVOKING** The postulates and theorems in this book represent Euclidean geometry. In spherical geometry, all points are points on the surface of a sphere. A line is a circle on the sphere whose diameter is equal to the diameter of the sphere. In spherical geometry, how many right angles are formed by two perpendicular lines? Justify your answer.

29. Construct a square of side length AB.

30. Inscribe a square in a circle with radius AB.

31. **ANALYZING RELATIONSHIPS** The painted line segments that form the path of a crosswalk are usually perpendicular to the crosswalk. Sketch what the segments in the photo would look like if they were perpendicular to the crosswalk. Which type of line segment requires less paint? Explain your reasoning.

32. **ABSTRACT REASONING** Two lines, a and b, are perpendicular to line c. Line d is parallel to line c. The distance between lines a and b is x meters. The distance between lines c and d is y meters. What shape is formed by the intersections of the four lines?

33. **WRITING** Describe how you would find the distance from a point to a plane. Can you find the distance from a line to a plane? Explain your reasoning.

---

**Maintaining Mathematical Proficiency**

Reviewing what you learned in previous grades and lessons

Simplify the ratio.  
(Skills Review Handbook)

34. \( \frac{6 - (-4)}{8 - 3} \)  
35. \( \frac{3 - 5}{4 - 1} \)  
36. \( \frac{8 - (-3)}{7 - (-2)} \)  
37. \( \frac{13 - 4}{2 - (-1)} \)

Find the slope and the y-intercept of the graph of the linear equation.  
(Section 3.5)

38. \( y = 3x + 9 \)  
39. \( y = -\frac{1}{2}x + 7 \)  
40. \( y = \frac{1}{6}x - 8 \)  
41. \( y = -8x - 6 \)
10.5 Using Parallel and Perpendicular Lines

Essential Question
How can you find the distance between two parallel lines?

Exploration 1
Finding the Distance Between Two Parallel Lines

Work with a partner.

a. Draw a line and label it \( \ell \).
   Draw a point not on line \( \ell \) and label it \( P \).

b. Construct a line through point \( P \) perpendicular to line \( \ell \).

c. Use a centimeter ruler to measure the distance from point \( P \) to line \( \ell \).

d. Construct a line through point \( P \) parallel to line \( \ell \) and label it \( m \).

e. Choose any point except point \( P \) on line \( \ell \) or line \( m \) and label it \( Q \). Describe how to find the distance from point \( Q \) to the other line.

f. Find the distance from point \( Q \) to the other line. Compare this distance to the distance from point \( P \) to line \( \ell \).

g. Is the distance from any point on line \( \ell \) to line \( m \) constant? Explain your reasoning.

Communicate Your Answer

2. How can you find the distance between two parallel lines?

3. Use centimeter graph paper and a centimeter ruler to find the distance between the two parallel lines.
   a. \( y = 2x + 2 \)
   b. \( y = -x + 4 \)
   c. \( y = 2x - 7 \)
   d. \( y = -x - 5 \)
What You Will Learn

- Prove the slope criteria for parallel lines.
- Find the distance from a point to a line.
- Find the distance between two parallel lines.

Proving the Slope Criteria for Parallel Lines

In the coordinate plane, the x-axis and the y-axis are perpendicular. Horizontal lines are parallel to the x-axis, and vertical lines are parallel to the y-axis.

Theorems

**Slopes of Parallel Lines**

In a coordinate plane, two distinct nonvertical lines are parallel if and only if they have the same slope.

Any two vertical lines are parallel.

*Proof* p. 528; Ex. 26, p. 532

**Slopes of Perpendicular Lines**

In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is $-1$.

Horizontal lines are perpendicular to vertical lines.

*Proof* Ex. 28, p. 532; p. 636

**CONNECTIONS TO ALGEBRA**

The information you learned about parallel and perpendicular lines in Section 4.3 can be stated as theorems.

**PROOF** Part of Slopes of Parallel Lines Theorem

*Given* Two distinct nonvertical lines in a coordinate plane

*Prove* The lines are parallel if and only if they have the same slope.

*Paragraph Proof* Consider the system of linear equations at the right, where Equations 1 and 2 represent two distinct nonvertical lines in a coordinate plane. The lines are parallel if and only if the system has no solution.

Substitute $m_2 x + b_2$ for $y$ in Equation 1.

$$y = m_1 x + b_1 \quad \text{Equation 1}$$

$$y = m_2 x + b_2 \quad \text{Equation 2}$$

$$m_2 x + b_2 = m_1 x + b_1 \quad \text{Substitute } m_2 x + b_2 \text{ for } y.$$  

$$m_2 x - m_1 x = b_1 - b_2 \quad \text{Isolate like terms.}$$

$$(m_2 - m_1) x = b_1 - b_2 \quad \text{Factor.}$$

When $m_1 \neq m_2$, you can divide each side of the equation above by $m_2 - m_1$ to find the value of $x$ and then substitute the value of $x$ into Equation 1 or 2 to find the value of $y$. So, the system has a solution when $m_1 \neq m_2$. When $m_1 = m_2$, the equation simplifies to $b_2 = b_1$ for all values of $x$. This is false because $b_1$ and $b_2$ must be different for the lines to be distinct. So, the system has no solution when $m_1 = m_2$. Because two distinct nonvertical lines are parallel if and only if their system has no solution and their system has no solution if and only if they have the same slope, two distinct nonvertical lines in a coordinate plane are parallel if and only if they have the same slope.
Finding the Distance from a Point to a Line

Recall that the distance from a point to a line is the length of the perpendicular segment from the point to the line.

**Example 1** Finding the Distance from a Point to a Line

Find the distance from the point (1, 0) to the line \( y = -x + 3 \).

**Solution**

**Step 1** Find an equation of the line perpendicular to the line \( y = -x + 3 \) that passes through the point (1, 0).

First, find the slope \( m \) of the perpendicular line. The line \( y = -x + 3 \) has a slope of \(-1\). Use the Slopes of Perpendicular Lines Theorem.

\[
-1 \cdot m = -1 \\
m = 1
\]

Then find the \( y \)-intercept \( b \) by using \( m = 1 \) and \((x, y) = (1, 0)\).

\[
y = mx + b \\
0 = 1(1) + b \\
-1 = b
\]

Because \( m = 1 \) and \( b = -1 \), an equation of the line is \( y = x - 1 \).

**Step 2** Use the two equations to write and solve a system of equations to find the point where the two lines intersect.

\[
y = -x + 3 \quad \text{Equation 1} \\
y = x - 1 \quad \text{Equation 2}
\]

Substitute \(-x + 3\) for \( y \) in Equation 2.

\[
y = x - 1 \quad \text{Equation 2} \\
-x + 3 = x - 1 \quad \text{Substitute } -x + 3 \text{ for } y.
\]

\[
x = 2 \quad \text{Solve for } x.
\]

Substitute 2 for \( x \) in Equation 1 and solve for \( y \).

\[
y = -x + 3 \quad \text{Equation 1} \\
y = -2 + 3 \quad \text{Substitute 2 for } x.
\]

\[
y = 1 \quad \text{Simplify.}
\]

So, the perpendicular lines intersect at (2, 1).

**Step 3** Use the Distance Formula to find the distance from (1, 0) to (2, 1).

\[
distance = \sqrt{(1 - 2)^2 + (0 - 1)^2} = 1.4
\]

So, the distance from the point (1, 0) to the line \( y = -x + 3 \) is about 1.4 units.

**Monitoring Progress**

1. Find the distance from the point (6, 4) to the line \( y = x + 4 \).
2. Find the distance from the point (−1, 6) to the line \( y = -2x \).
Finding the Distance Between Two Parallel Lines

The distance between two parallel lines is the length of any perpendicular segment joining the two lines. For instance, the distance between line \( p \) and line \( m \) below is \( CD \) or \( EF \).

**EXAMPLE 2** Finding the Distance Between Two Parallel Lines

Find the distance between \( y = \frac{1}{2}x - 2 \) and \( y = \frac{1}{2}x + 1 \).

**SOLUTION**

**Step 1** Find an equation of a line perpendicular to the two parallel lines. The slopes of the two parallel lines are both \( \frac{1}{2} \). Use the Slopes of Perpendicular Lines Theorem.

\[
\frac{1}{2} \cdot m = -1
\]

The product of the slopes of \( \perp \) lines is \(-1\).

\[
m = -2
\]

Multiply each side by 2.

Any line with a slope of \(-2\) is perpendicular to the two parallel lines. Use \( y = -2x - 2 \) because it has the same \( y \)-intercept, \((0, -2)\), as one of the two parallel lines.

**Step 2** The distance between the points where \( y = -2x - 2 \) intersects the parallel lines is the distance between the lines. You already know that one point of intersection is \((0, -2)\). Use substitution to find the other point of intersection.

\[
y = \frac{1}{2}x + 1 \quad \text{Write original equation.}
\]

\[
-2x - 2 = \frac{1}{2}x + 1 \quad \text{Substitute } -2x - 2 \text{ for } y.
\]

\[
x = -\frac{6}{5} \quad \text{Solve for } x.
\]

Substituting \( x = -\frac{6}{5} \) into \( y = -2x - 2 \) and solving for \( y \) gives the \( y \)-value of the other point of intersection, which is \( \frac{2}{5} \). So, the points of intersection are \((0, -2)\) and \((\frac{-6}{5}, \frac{2}{5})\).

**Step 3** Use the Distance Formula to find the distance from \((0, -2)\) to \((\frac{-6}{5}, \frac{2}{5})\).

\[
\text{distance} = \sqrt{\left(\frac{-6}{5} - 0\right)^2 + \left(\frac{2}{5} - (-2)\right)^2} \approx 2.7
\]

So, the distance between \( y = \frac{1}{2}x - 2 \) and \( y = \frac{1}{2}x + 1 \) is about 2.7 units.

**Monitoring Progress**

Help in English and Spanish at BigIdeasMath.com

Find the distance between the parallel lines.

3. \( y = 2x - 4, y = 2x + 1 \)
4. \( y = -3x - 2, y = -3x - 4 \)
5. \( y = \frac{3}{4}x - 3, y = \frac{3}{4}x + 2 \)
6. \( y = -\frac{3}{2}x - 6, y = -\frac{3}{2}x + 5 \)
In Exercises 3–8, find the distance from point A to the given line. (See Example 1.)

3. A(−1, 7), y = 3x

4. A(6, −2), y = −4x

5. A(−9, −3), y = x − 6

6. A(−4, 4), y = −2x + 1

7. A(−1, 4), line with a slope of −3 that passes through (2, −4)

8. A(15, −21), line for which f(4) = −8 and f(8) = −18

In Exercises 9–14, find the distance between the parallel lines. (See Example 2.)

9. y = 2x + 7, y = −4x + 4

10. y = −3x + 1, y = −3x + 4

11. y = −4x + 5, y = −4x + 7

12. y = 3x + 6, y = 3x − 2

13. y = −\frac{2}{3}x + 8, parallel line that passes through (0, 0)

14. y = −\frac{5}{6}x − 1, parallel line that passes through (6, −4)

In Exercises 15–18, use the map shown. Each unit in the coordinate plane corresponds to 1 mile.

15. Find the distance from the school to Cherry Street.

16. Find the distance from the stadium to Pine Avenue.

17. Find the distance between Oak Avenue and Pine Avenue.

18. Find the distance between Hazel Street and Cherry Street.

19. **PROBLEM SOLVING** A gazebo is being built near a nature trail. An equation of the line representing the nature trail is \( y = \frac{1}{3}x − 4 \). Each unit in the coordinate plane corresponds to 10 feet. Approximately how far is the gazebo from the nature trail?
20. **PROBLEM SOLVING** A bike path is being constructed perpendicular to Washington Boulevard starting at point $P(2, 2)$. An equation of the line representing Washington Boulevard is $y = -\frac{2}{3}x$. Each unit in the coordinate plane corresponds to 10 feet. Approximately how far is the starting point from Washington Boulevard?

21. **ERROR ANALYSIS** Describe and correct the error in finding the distance between $y = x + 2$ and $y = x + 1$.

22. **HOW DO YOU SEE IT?** Determine whether quadrilateral $JKLM$ is a square. Explain your reasoning.

23. **MAKING AN ARGUMENT** Your classmate claims that no two nonvertical parallel lines can have the same $y$-intercept. Is your classmate correct? Explain.

24. **PROVING A THEOREM** In Exercises 24 and 25, use the slopes of lines to write a paragraph proof of the theorem.

25. **Transitive Property of Parallel Lines Theorem** If two lines are parallel to the same line, then they are parallel to each other.

26. **PROOF** Prove the statement: If two lines are vertical, then they are parallel.

27. **PROOF** Prove the statement: If two lines are horizontal, then they are parallel.

28. **PROOF** Prove that horizontal lines are perpendicular to vertical lines.

29. **MATHEMATICAL CONNECTIONS** Find the area of $\triangle ABC$.

30. **THOUGHT PROVOKING** Find a formula for the distance from the point $(x_0, y_0)$ to the line $ax + by = 0$. Verify your formula using a point and a line.

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**Maintaining Mathematical Proficiency**

Plot the point in a coordinate plane. *(Skills Review Handbook)*

31. $A(3, 6)$
32. $B(0, -4)$
33. $C(5, 0)$
34. $D(-1, -2)$

Copy and complete the table. *(Skills Review Handbook)*

35. $\begin{array}{c|c|c|c|c|c} x & -2 & -1 & 0 & 1 & 2 \\ \hline y = x + 9 & & & & & \\ \end{array}$
36. $\begin{array}{c|c|c|c|c|c} x & -2 & -1 & 0 & 1 & 2 \\ \hline y = x - \frac{3}{4} & & & & & \\ \end{array}$
10.4–10.5  What Did You Learn?

Core Vocabulary

distance from a point to a line, p. 520
perpendicular bisector, p. 521

Core Concepts

Section 10.4
Finding the Distance from a Point to a Line, p. 520
Constructing Perpendicular Lines, p. 521
Linear Pair Perpendicular Theorem, p. 522
Perpendicular Transversal Theorem, p. 522
Lines Perpendicular to a Transversal Theorem, p. 522

Section 10.5
Slopes of Parallel Lines, p. 528
Slopes of Perpendicular Lines, p. 528
Finding the Distance from a Point to a Line, p. 529
Finding the Distance Between Two Parallel Lines, p. 530

Mathematical Practices

1. Compare the effectiveness of the argument in Exercise 24 on page 525 with the argument “You can find the distance between any two parallel lines.” What flaw(s) exist in the argument(s)? Does either argument use correct reasoning? Explain.

2. Look back at your construction of a square in Exercise 29 on page 526. How would your construction change if you were to construct a rectangle?

3. In Exercise 23 on page 532, another classmate makes the same claim about the x-intercept. Respond to your classmate’s argument by adapting your original answer.

Performance Task:

Squaring a Treehouse

When builders construct any structure, they make sure it is plumb, level, and square. What do these terms mean? How are they related to the concepts of geometry? What relationships between them can you support?

To explore the answers to these questions and more, check out the Performance Task and Real-Life STEM video at BigIdeasMath.com.
10.1 Pairs of Lines and Angles  (pp. 497–502)

Think of each segment in the figure as part of a line.

a. Which line(s) appear perpendicular to $\overrightarrow{AB}$?
   - $BD, AC, BH$, and $AG$ appear perpendicular to $\overrightarrow{AB}$.

b. Which line(s) appear parallel to $\overrightarrow{AB}$?
   - $CD, GH$, and $EF$ appear parallel to $\overrightarrow{AB}$.

c. Which line(s) appear skew to $\overrightarrow{AB}$?
   - $CF, CE, DF, FH$, and $EG$ appear skew to $\overrightarrow{AB}$.

d. Which plane(s) appear parallel to plane $ABC$?
   - Plane $EFG$ appears parallel to plane $ABC$.

Think of each segment in the figure as part of a line. Which line(s) or plane(s) appear to fit the description?

1. line(s) perpendicular to $\overrightarrow{QR}$
2. line(s) parallel to $\overrightarrow{QR}$
3. line(s) skew to $\overrightarrow{QR}$
4. plane(s) parallel to plane $LMQ$

10.2 Parallel Lines and Transversals  (pp. 503–508)

Find the value of $x$.

By the Vertical Angles Congruence Theorem, $m\angle 6 = 50^\circ$.

$$\begin{align*}
(x - 5)^\circ + m\angle 6 &= 180^\circ \\
(x - 5)^\circ + 50^\circ &= 180^\circ \\
x + 45 &= 180 \\
x &= 135
\end{align*}$$

So, the value of $x$ is 135.

Find the values of $x$ and $y$.

5. $x^\circ, 35^\circ, y^\circ$
6. $x^\circ, 48^\circ, y^\circ$
7. $2x^\circ, 58^\circ, 2y^\circ$
8. $(5x - 21)^\circ, (6x + 32)^\circ$
**10.3 Proofs with Parallel Lines** (pp. 509–516)

Find the value of \( x \) that makes \( m \parallel n \).

By the Alternate Interior Angles Converse, \( m \parallel n \) when the marked angles are congruent.

\[
(5x + 8)^\circ = 53^\circ \\
5x = 45 \\
x = 9
\]

\( \rightarrow \) The lines \( m \) and \( n \) are parallel when \( x = 9 \).

Find the value of \( x \) that makes \( m \parallel n \).

9. \( x^\circ \) \( \underline{73^\circ} \) \( \underline{m} \) \( \underline{n} \)

10. \( \underline{147^\circ} \) \( \underline{(x + 14)^\circ} \) \( \underline{m} \) \( \underline{n} \)

11. \( (2x + 20)^\circ \) \( \underline{3x^\circ} \) \( \underline{m} \) \( \underline{n} \)

12. \( \underline{m} \) \( \underline{(7x - 11)^\circ} \) \( \underline{n} \) \( \underline{(4x + 58)^\circ} \)

**10.4 Proofs with Perpendicular Lines** (pp. 519–526)

Determine which lines, if any, must be parallel. Explain your reasoning.

Lines \( a \) and \( b \) are both perpendicular to \( d \), so by the Lines Perpendicular to a Transversal Theorem, \( a \parallel b \).

Also, lines \( c \) and \( d \) are both perpendicular to \( b \), so by the Lines Perpendicular to a Transversal Theorem, \( c \parallel d \).

Determine which lines, if any, must be parallel. Explain your reasoning.

13. \( \underline{x} \) \( \underline{y} \) \( \underline{z} \) 

14. \( \underline{x} \) \( \underline{w} \) \( \underline{z} \) 

15. \( a \) \( \underline{m} \) \( \underline{n} \) \( \underline{b} \) 

16. \( a \) \( \underline{m} \) \( \underline{n} \) \( \underline{b} \) \( \underline{c} \)
Using Parallel and Perpendicular Lines  

Find the distance from the point \((4, 0)\) to the line \(y = \frac{1}{2}x + 1\).

**Step 1**  
Find an equation of the line perpendicular to the line \(y = \frac{1}{2}x + 1\) that passes through the point \((4, 0)\).

First, find the slope \(m\) of the perpendicular line. The line \(y = \frac{1}{2}x + 1\) has a slope of \(\frac{1}{2}\). By the Slopes of Perpendicular Lines Theorem, the slope of the perpendicular line is \(m = -2\).

Then find the \(y\)-intercept \(b\) by using \(m = -2\) and \((x, y) = (4, 0)\).

\[
\begin{align*}
\frac{1}{2}x + 1 &= \frac{1}{2}x + 1 \\
0 &= -2(4) + b \\
8 &= b
\end{align*}
\]

Because \(m = -2\) and \(b = 8\), an equation of the line is \(y = -2x + 8\).

**Step 2**  
Use the two equations to write and solve a system of equations to find the point where the two lines intersect.

\[
\begin{align*}
y &= \frac{1}{2}x + 1 & \text{Equation 1} \\
y &= -2x + 8 & \text{Equation 2}
\end{align*}
\]

Substitute \(\frac{1}{2}x + 1\) for \(y\) in Equation 2.

\[
\frac{1}{2}x + 1 = -2x + 8
\]

Substitute \(\frac{14}{5}\) for \(x\) in Equation 1 and solve for \(y\).

\[
\begin{align*}
y &= \frac{1}{2}x + 1 & \text{Equation 1} \\
y &= \frac{1}{2}(\frac{14}{5}) + 1 & \text{Substitute } \frac{14}{5} \text{ for } x. \\
y &= \frac{12}{5} & \text{Simplify.}
\end{align*}
\]

So, the perpendicular lines intersect at \(\left(\frac{14}{5}, \frac{12}{5}\right)\).

**Step 3**  
Use the Distance Formula to find the distance from \((4, 0)\) to \(\left(\frac{14}{5}, \frac{12}{5}\right)\).

\[
\text{distance} = \sqrt{\left(\frac{14}{5} - 4\right)^2 + \left(\frac{12}{5} - 0\right)^2} \approx 2.7
\]

So, the distance from the point \((4, 0)\) to the line \(y = \frac{1}{2}x + 1\) is about 2.7 units.

Find the distance from point \(A\) to the given line.

17. \(A(2, -1), y = -x + 4\)  
18. \(A(-6, 4), y = -\frac{3}{4}x + 1\)

Find the distance between the parallel lines.

19. \(y = 6x - 4 \text{ and } y = 6x + 10\)  
20. \(y = -\frac{1}{4}x - 3 \text{ and } y = -\frac{1}{4}x + 4\)
Find the values of $x$ and $y$. State which theorem(s) you used.

1. $61^\circ \quad x \quad y$ 
2. $(11y + 19)^\circ \quad 8x^\circ \quad 96^\circ$ 
3. $42^\circ \quad (8x + 2)^\circ \quad (6(2y - 3))^\circ$

Find the distance from point $A$ to the given line.

4. $A(3, 4), y = -x$
5. $A(-3, 7), y = \frac{1}{3}x - 2$

Find the value of $x$ that makes $m \parallel n$.

6. 
7. $(4x + 24)^\circ \quad 8x^\circ$
8. $(11x + 33)^\circ \quad (6x - 6)^\circ$

Find the distance between the parallel lines.

9. $y = -5x - 14, y = -5x + 1$
10. $y = \frac{2}{3}x - 9, y = \frac{2}{3}x + 7$

11. A student says, “Because $j \perp k, j \perp l$,” What missing information is the student assuming from the diagram? Which theorem is the student trying to use?

12. You and your family are visiting some attractions while on vacation. You and your mom visit the shopping mall while your dad and your sister visit the aquarium. You decide to meet at the intersection of lines $q$ and $p$. Each unit in the coordinate plane corresponds to 50 yards.
   a. Find an equation of line $q$.
   b. Find an equation of line $p$.
   c. What are the coordinates of the meeting point?
   d. What is the distance from the meeting point to the subway?

13. Identify an example on the puzzle cube of each description. Explain your reasoning.
   a. a pair of skew lines
   b. a pair of perpendicular lines
   c. a pair of parallel lines
   d. a pair of congruent corresponding angles
   e. a pair of congruent alternate interior angles
Cumulative Assessment

1. Use the steps in the construction to explain how you know that \( \overline{CD} \) is the perpendicular bisector of \( \overline{AB} \).

   **Step 1**
   
   **Step 2**
   
   **Step 3**

2. The table shows the distances you travel over a 6-hour period. Create an equation that models the distance traveled as a function of the number of hours.

<table>
<thead>
<tr>
<th>Hours, ( x )</th>
<th>Distance (miles), ( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>62</td>
</tr>
<tr>
<td>2</td>
<td>123</td>
</tr>
<tr>
<td>3</td>
<td>184</td>
</tr>
<tr>
<td>4</td>
<td>245</td>
</tr>
<tr>
<td>5</td>
<td>306</td>
</tr>
<tr>
<td>6</td>
<td>367</td>
</tr>
</tbody>
</table>

3. Classify each pair of angles whose measurements are given.

   a. \( \overline{AB} \)

   b. \( \overline{FG} \)

   c. \( \overline{JK} \)

   d. \( \overline{MN} \)

4. Your school is installing new turf on the football field. A coordinate plane has been superimposed on a diagram of the football field where 1 unit = 20 feet.

   a. What is the length of the field?

   b. What is the perimeter of the field?

   c. Turf costs $2.69 per square foot. Your school has a $150,000 budget. Does the school have enough money to purchase new turf for the entire field?
5. The graph shows the function \( f(x) = 2(3)^x \).
   a. Is the function increasing or decreasing for increasing values of \( x \)?
   b. Identify any \( x \)- and \( y \)-intercepts.

6. Which of the following is true when \( \overrightarrow{AB} \) and \( \overrightarrow{CD} \) are skew?
   - \( A \) \( \overrightarrow{AB} \) and \( \overrightarrow{CD} \) are parallel.
   - \( B \) \( \overrightarrow{AB} \) and \( \overrightarrow{CD} \) intersect.
   - \( C \) \( \overrightarrow{AB} \) and \( \overrightarrow{CD} \) are perpendicular.
   - \( D \) \( A, B, \) and \( C \) are noncollinear.

7. Select all the statements that are valid for the data in the two-way table.

<table>
<thead>
<tr>
<th>Class</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freshman</td>
<td>28</td>
<td>36</td>
</tr>
<tr>
<td>Sophomore</td>
<td>22</td>
<td>14</td>
</tr>
</tbody>
</table>

Of the students who are attending, 56% are freshmen.

Of the students who are not attending, 28% are sophomores.

Of the students who are attending, about 64% are freshmen.

Of the students who are sophomores, about 50% are not attending.

8. You and your friend walk to school together every day. You meet at the halfway point between your houses first and then walk to school. Each unit in the coordinate plane corresponds to 50 yards.
   a. What are the coordinates of the midpoint of the line segment joining the two houses?
   b. What is the distance that the two of you walk together?

9. Which of the following is not true for the diagram shown?
   - \( A \) \( \angle ABF \) and \( \angle EFH \) are corresponding angles.
   - \( B \) \( \angle DBC \) and \( \angle HFE \) are alternate exterior angles.
   - \( C \) \( \angle ABF \) and \( \angle CBF \) are alternate interior angles.
   - \( D \) \( \angle ABF \) and \( \angle EFB \) are consecutive interior angles.