

Algebra: Real Numbers and the Pythagorean Theorem

“How far can you see from a tall building?”

The Sears Tower in Chicago is 1,450 feet high. You can determine approximately how far you can see from the top of the Sears Tower by multiplying 1.23 by $\sqrt{1,450}$. The symbol $\sqrt{1,450}$ represents the square root of 1,450.

You will solve problems about how far a person can see from a given height in Lesson 3-3.

GETTING STARTED

► Diagnose Readiness

Take this quiz to see if you are ready to begin Chapter 3. Refer to the lesson or page number in parentheses for review.

Vocabulary Review

State whether each sentence is *true* or *false*. If *false*, replace the underlined word to make a true sentence.

- The number 0.6 is a rational number. (Lesson 2-1)
- In the number 3^2 , the base is 2. (Lesson 2-8)

Prerequisite Skills

Graph each point on a coordinate plane. (Page 614)

- $A(-1, 3)$
- $B(2, -4)$
- $C(-2, -3)$
- $D(-4, 0)$

Evaluate each expression. (Lesson 1-2)

- $2^2 + 4^2$
- $3^2 + 3^2$
- $10^2 + 8^2$
- $7^2 + 5^2$

Solve each equation. Check your solution. (Lesson 1-8)

- $x + 13 = 45$
- $56 + d = 71$
- $101 = 39 + a$
- $62 = 45 + m$

Express each decimal as a fraction in simplest form. (Lesson 2-1)

- $0.\bar{6}$
- 0.35
- 0.375

Between which two of the following numbers does each number lie?

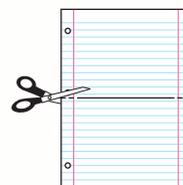
- 1, 4, 9, 16, 25, 36, 49, 64, 81 (Lesson 2-2)
- 38
- 74

FOLDABLES Study Organizer

Real Numbers and the Pythagorean Theorem
Make this Foldable to help you organize your notes. Begin with two sheets of $8\frac{1}{2}$ " by 11" paper.

STEP 1 Fold and Cut One Sheet

Fold in half from top to bottom. Cut along fold from edges to margin.



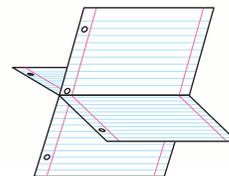
STEP 2 Fold and Cut the Other Sheet

Fold in half from top to bottom. Cut along fold between margins.



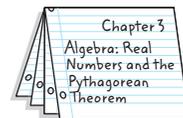
STEP 3 Assemble

Insert first sheet through second sheet and align folds.



STEP 4 Label

Label each page with a lesson number and title.



Noteables™

Chapter Notes Each time you find this logo throughout the chapter, use your *Noteables™*: *Interactive Study Notebook with Foldables™* or your own notebook to take notes. Begin your chapter notes with this Foldable activity.



Readiness To prepare yourself for this chapter with another quiz, visit msmath3.net/chapter_readiness

3-1

Square Roots

HANDS-ON Mini Lab

Materials

- color tiles



Work with a partner.

Look at the two square arrangements of tiles at the right. Continue this pattern of square arrays until you reach 5 tiles on each side.

1. Copy and complete the following table.

Tiles on a Side	1	2	3	4	5
Total Number of Tiles in the Square Arrangement	1	4			

2. Suppose a square arrangement has 36 tiles. How many tiles are on a side?
3. What is the relationship between the number of tiles on a side and the number of tiles in the arrangement?

What You'll LEARN

Find square roots of perfect squares.

NEW Vocabulary

perfect square
square root
radical sign
principal square root

REVIEW Vocabulary

exponent: tells the number of times the base is used as a factor
(Lesson 1-7)

Numbers such as 1, 4, 9, 16, and 25 are called **perfect squares** because they are squares of whole numbers. The opposite of squaring a number is finding a **square root**.

Noteables™

Key Concept: Square Root

Words A square root of a number is one of its two equal factors.

Symbols Arithmetic

Since $3 \cdot 3 = 9$, a square root of 9 is 3.

Since $(-3)(-3) = 9$, a square root of 9 is -3 .

Algebra

If $x^2 = y$, then x is a square root of y .

The symbol $\sqrt{\quad}$, called a **radical sign**, is used to indicate the positive square root. The symbol $-\sqrt{\quad}$ is used to indicate the negative square root.

READING in the Content Area

For strategies in reading this lesson, visit msmath3.net/reading.

EXAMPLE Find a Square Root

- 1 Find $\sqrt{64}$.

$\sqrt{64}$ indicates the *positive* square root of 64.

Since $8^2 = 64$, $\sqrt{64} = 8$.

EXAMPLE**Find the Negative Square Root****1** Find $-\sqrt{121}$. $-\sqrt{121}$ indicates the *negative* square root of 121.Since $(-11)(-11) = 121$, $-\sqrt{121} = -11$.**Your Turn** Find each square root.

a. $\sqrt{49}$

b. $-\sqrt{225}$

c. $-\sqrt{0.16}$

READING Math**Square Roots** A positive square root is called the principal square root.**EXAMPLE****Use Square Roots to Solve an Equation****3** **ALGEBRA** Solve $t^2 = \frac{25}{36}$.

$$t^2 = \frac{25}{36}$$

Write the equation.

$$\sqrt{t^2} = \sqrt{\frac{25}{36}} \text{ or } -\sqrt{\frac{25}{36}}$$

Take the square root of each side.

$$t = \frac{5}{6} \text{ or } -\frac{5}{6}$$

Notice that $\frac{5}{6} \cdot \frac{5}{6} = \frac{25}{36}$ and $(-\frac{5}{6})(-\frac{5}{6}) = \frac{25}{36}$.The equation has two solutions, $\frac{5}{6}$ and $-\frac{5}{6}$.**Your Turn** Solve each equation.

d. $y^2 = \frac{4}{25}$

e. $196 = a^2$

f. $m^2 = 0.09$

In real-life situations, a negative answer may not make sense.

EXAMPLE**Use an Equation to Solve a Problem****4** **HISTORY** The Great Pyramid of Giza has a square base with an area of about 567,009 square feet. Determine the length of each side of its base.**STUDY TIP****Rational Exponents**Exponents can also be used to indicate the square root. $9^{\frac{1}{2}}$ means the same thing as $\sqrt{9}$. $9^{\frac{1}{2}}$ is read *nine to the one half power*. $9^{\frac{1}{2}} = 3$.

Words	Area	is equal to	the square of the length of a side.
Variables	A	=	s^2
Equation	567,009	=	s^2

$$567,009 = s^2$$

Write the equation.

$$\sqrt{567,009} = \sqrt{s^2}$$

Take the square root of each side.

$$\boxed{2nd} \sqrt{} 567009 \boxed{ENTER}$$

Use a calculator.

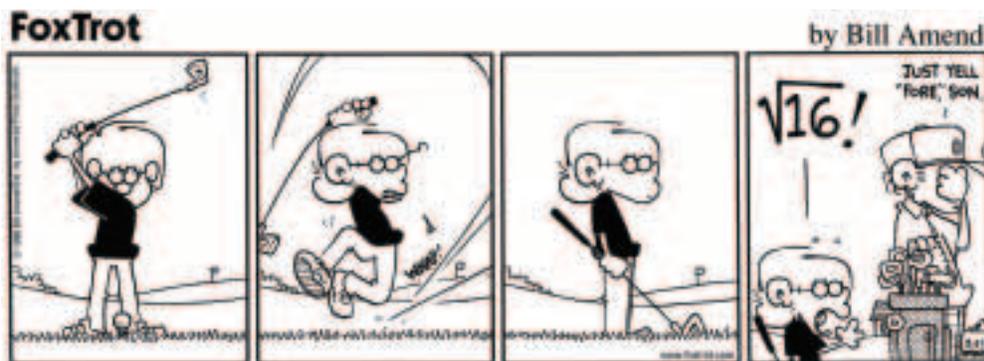
$$753 \text{ or } -753 = s$$

The length of a side of the base of the Great Pyramid of Giza is about 753 feet since distance cannot be negative.



Skill and Concept Check

1. **Writing Math** Explain the meaning of $\sqrt{16}$ in the cartoon.



2. Write the symbol for the negative square root of 25.
3. **OPEN ENDED** Write an equation that can be solved by taking the square root of a perfect square.
4. **FIND THE ERROR** Diana and Terrell are solving the equation $x^2 = 81$. Who is correct? Explain.

Diana
 $x^2 = 81$
 $x = 9$

Terrell
 $x^2 = 81$
 $x = 9$ or $x = -9$

GUIDED PRACTICE

Find each square root.

5. $\sqrt{25}$ 6. $-\sqrt{100}$ 7. $-\sqrt{\frac{16}{81}}$ 8. 0.64

ALGEBRA Solve each equation.

9. $p^2 = 36$ 10. $n^2 = 169$ 11. $900 = r^2$ 12. $t^2 = \frac{1}{9}$

13. **ALGEBRA** If $n^2 = 256$, find n .

Practice and Applications

Find each square root.

14. $\sqrt{16}$ 15. $\sqrt{81}$ 16. $-\sqrt{64}$
 17. $-\sqrt{36}$ 18. $-\sqrt{196}$ 19. $-\sqrt{144}$
 20. $\sqrt{256}$ 21. $\sqrt{324}$ 22. $-\sqrt{\frac{16}{25}}$
 23. $-\sqrt{\frac{9}{49}}$ 24. $\sqrt{0.25}$ 25. $\sqrt{1.44}$

26. Find the positive square root of 169.
 27. What is the negative square root of 400?

HOMEWORK HELP

For Exercises	See Examples
14–27	1, 2
28–41	3
42–45	4

Extra Practice
 See pages 622, 650.

ALGEBRA Solve each equation.

28. $v^2 = 81$ 29. $b^2 = 100$ 30. $y^2 = 225$ 31. $s^2 = 144$
32. $1,600 = a^2$ 33. $2,500 = d^2$ 34. $w^2 = 625$ 35. $m^2 = 961$
36. $\frac{25}{81} = p^2$ 37. $\frac{9}{64} = c^2$ 38. $r^2 = 2.25$ 39. $d^2 = 1.21$

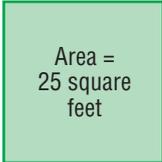
40. **ALGEBRA** Find a number that when squared equals 1.0404.

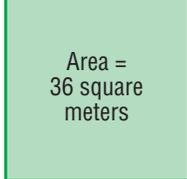
41. **ALGEBRA** Find a number that when squared equals 4.0401.

42. **MARCHING BAND** A marching band wants to make a square formation. If there are 81 members in the band, how many should be in each row?

GEOMETRY The formula for the perimeter of a square is $P = 4s$, where s is the length of a side. Find the perimeter of each square.

43. 

44. 

45. 

46. **MULTI STEP** Describe three different-sized squares that you could make at the same time out of 130 square tiles. How many tiles are left?

47. **CRITICAL THINKING** Find each value.

- a. $(\sqrt{36})^2$ b. $(\sqrt{81})^2$ c. $(\sqrt{21})^2$ d. $(\sqrt{x})^2$

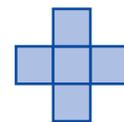
48. **CRITICAL THINKING** True or False? $\sqrt{-25} = -5$. Explain.

Spiral Review with Standardized Test Practice 

49. **MULTIPLE CHOICE** What is the solution of $a^2 = 49$?

- A -7 B 7 C 7 or -7 D 7 or 0 or -7

50. **SHORT RESPONSE** The area of each square is 4 square units. Find the perimeter of the figure.



51. **SPACE** The Alpha Centauri stars are about 2.5×10^{13} miles from Earth. Write this distance in standard form. (Lesson 2-9)

Write each expression using exponents. (Lesson 2-8)

52. $6 \cdot 6 \cdot 6$ 53. $2 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 2$ 54. $a \cdot a \cdot a \cdot b$ 55. $s \cdot t \cdot t \cdot s \cdot s \cdot t \cdot s$

56. What is the absolute value of -18 ? (Lesson 1-3)

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL Between which two perfect squares does each number lie? (Lesson 2-2)

57. 57 58. 68 59. 33 60. 40

3-2

Estimating Square Roots

HANDS-ON Mini Lab

Materials

- grid paper

What You'll LEARN

Estimate square roots.

MATH Symbols

\approx about equal to

Work with a partner.

On grid paper, draw the largest possible square using no more than 40 small squares.

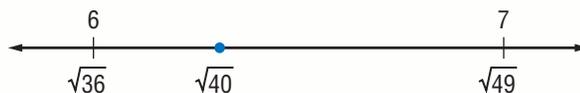
On grid paper, draw the smallest possible square using at least 40 small squares.

1. How many squares are on each side of the largest possible square using no more than 40 small squares?
2. How many squares are on each side of the smallest possible square using at least 40 small squares?
3. The value of $\sqrt{40}$ is between two consecutive whole numbers. What are the numbers?

Use grid paper to determine between which two consecutive whole numbers each value is located.

4. $\sqrt{23}$
5. $\sqrt{52}$
6. $\sqrt{27}$
7. $\sqrt{18}$

Since 40 is not a perfect square, $\sqrt{40}$ is not a whole number.



The number line shows that $\sqrt{40}$ is between 6 and 7. Since 40 is closer to 36 than 49, the best whole number estimate for $\sqrt{40}$ is 6.

EXAMPLE Estimate Square Roots

Estimate to the nearest whole number.

1 $\sqrt{83}$

- The first perfect square less than 83 is 81.
- The first perfect square greater than 83 is 100.

$$81 < 83 < 100 \quad \text{Write an inequality.}$$

$$9^2 < 83 < 10^2 \quad 81 = 9^2 \text{ and } 100 = 10^2$$

$$\sqrt{9^2} < \sqrt{83} < \sqrt{10^2} \quad \text{Take the square root of each number.}$$

$$9 < \sqrt{83} < 10 \quad \text{Simplify.}$$

So, $\sqrt{83}$ is between 9 and 10. Since 83 is closer to 81 than 100, the best whole number estimate for $\sqrt{83}$ is 9.

Your Turn Estimate to the nearest whole number.

- a. $\sqrt{35}$
- b. $\sqrt{170}$
- c. $\sqrt{14.8}$

Practice and Applications

Estimate to the nearest whole number.

- | | | | |
|-------------------|-------------------|--------------------|-------------------|
| 10. $\sqrt{11}$ | 11. $\sqrt{15}$ | 12. $\sqrt{44}$ | 13. $\sqrt{23}$ |
| 14. $\sqrt{113}$ | 15. $\sqrt{105}$ | 16. $\sqrt{82}$ | 17. $\sqrt{50}$ |
| 18. $\sqrt{15.6}$ | 19. $\sqrt{23.5}$ | 20. $\sqrt{85.1}$ | 21. $\sqrt{38.4}$ |
| 22. $\sqrt{200}$ | 23. $\sqrt{170}$ | 24. $\sqrt{150}$ | 25. $\sqrt{130}$ |
| 26. $\sqrt{630}$ | 27. $\sqrt{925}$ | 28. $\sqrt{1,300}$ | 29. $\sqrt{780}$ |

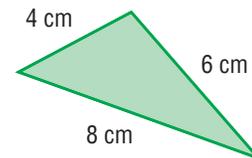
30. **ALGEBRA** Estimate the solution of $y^2 = 55$ to the nearest integer.

31. **ALGEBRA** Estimate the solution of $d^2 = 95$ to the nearest integer.

32. Order 7, 9, $\sqrt{50}$, and $\sqrt{85}$ from least to greatest.

33. Order $\sqrt{91}$, 7, 5, $\sqrt{38}$ from least to greatest.

34. **HISTORY** The Egyptian mathematician Heron created the formula $A = \sqrt{s(s-a)(s-b)(s-c)}$ to find the area A of a triangle. In this formula, a , b , and c are the measures of the sides, and s is one-half of the perimeter. Use this formula to estimate the area of the triangle.



35. **SCIENCE** The formula $t = \frac{\sqrt{h}}{4}$ represents the time t in seconds that it takes an object to fall from a height of h feet. If a ball is dropped from a height of 200 feet, estimate how long will it take to reach the ground.

36. **CRITICAL THINKING** If $x^3 = y$, then x is the cube root of y . Explain how to estimate the cube root of 30. What is the cube root of 30 to the nearest whole number?

Spiral Review with Standardized Test Practice

37. **MULTIPLE CHOICE** Which is the best estimate of the value of $\sqrt{54}$?

- (A) 6 (B) 7 (C) 8 (D) 27

38. **MULTIPLE CHOICE** If $x^2 = 38$, then a value of x is approximately

- (F) 5. (G) 6. (H) 7. (I) 24.

39. **ALGEBRA** Find a number that, when squared, equals 8,100. (Lesson 3-1)

40. **GEOGRAPHY** The Great Lakes cover about 94,000 square miles. Write this number in scientific notation. (Lesson 2-9)

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL Express each decimal as a fraction in simplest form. (Lesson 2-1)

41. 0.15

42. 0.8

43. $0.\bar{3}$

44. $0.\bar{4}$

HOMEWORK HELP

For Exercises See Examples

10–31 1

34–35 2

Extra Practice
See pages 622, 650.



3-3a

Problem-Solving Strategy

A Preview of Lesson 3-3

Use a Venn Diagram

What You'll LEARN

Solve problems using a Venn diagram.

Of the 12 students who ate lunch with me today, 9 are involved in music activities and 6 play sports. Four are involved in both music and sports.

How could we organize that information?

Explore

We know how many students are involved in each activity and how many are involved in both activities. We want to organize the information.

Plan

Let's use a **Venn diagram** to organize the information.

Solve

Draw two overlapping circles to represent the two different activities. Since 4 students are involved in both activities, place a 4 in the section that is part of both circles. Use subtraction to determine the number for each other section.

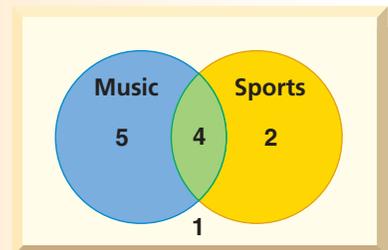
$$\text{only music: } 9 - 4 = 5$$

$$\text{only sports: } 6 - 4 = 2$$

$$\text{neither music nor sports: } 12 - 5 - 2 - 4 = 1$$

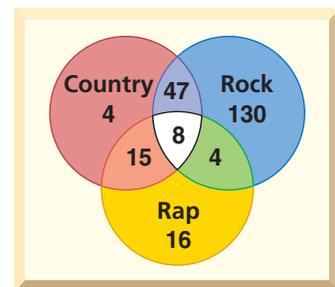
Examine

Check each circle to see if the appropriate number of students is represented.



Analyze the Strategy

1. **Tell** what each section of the Venn diagram above represents and the number of students that belong to that category.
2. **Use** the Venn diagram above to determine the number of students who are in either music or sports but not both.
3. **Write** a situation that can be represented by the Venn diagram at the right.



Apply the Strategy

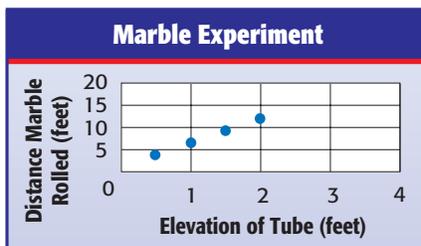
Solve. Use a Venn diagram.

4. **MARKETING** A survey showed that 83 customers bought wheat cereal, 83 bought rice cereal, and 20 bought corn cereal. Of those who bought exactly two boxes of cereal, 6 bought corn and wheat, 10 bought rice and corn, and 12 bought rice and wheat. Four customers bought all three. How many customers bought only rice cereal?
5. **FOOD** Napoli's Pizza conducted a survey of 75 customers. The results showed that 35 customers liked mushroom pizza, 41 liked pepperoni, and 11 liked both mushroom and pepperoni pizza. How many liked neither mushroom nor pepperoni pizza?

Mixed Problem Solving

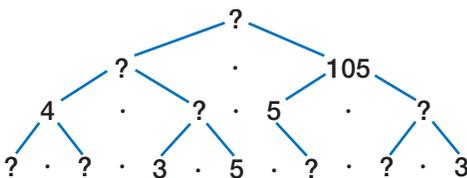
Solve. Use any strategy.

6. **SCIENCE** Emilio created a graph of data he collected for a science project. If the pattern continues, about how far will the marble roll if the end of the tube is raised to an elevation of $3\frac{1}{2}$ feet?



7. **MULTI STEP** Three after-school jobs are posted on the job board. The first job pays \$5.15 per hour for 15 hours of work each week. The second job pays \$10.95 per day for 2 hours of work, 5 days a week. The third job pays \$82.50 for 15 hours of work each week. If you want to apply for the best-paying job, which job should you choose? Explain your reasoning.

8. **FACTOR TREE** Copy and complete the factor tree.



9. **NUMBER THEORY** A subset is a part of a set. The symbol \subset means "is a subset of." Consider the following two statements.

integers \subset rational numbers
 rational numbers \subset integers

Are both statements true? Draw a Venn diagram to justify your answer.

HEALTH For Exercises 10 and 11, use the following information.

Dr. Bagenstose is an allergist. Her patients had the following symptoms last week.

Symptom(s)	Number of Patients
runny nose	22
watery eyes	20
sneezing	28
runny nose and watery eyes	8
runny nose and sneezing	15
watery eyes and sneezing	12
runny nose, watery eyes, and sneezing	5

10. Draw a Venn diagram of the data.
11. How many patients had only watery eyes?

12. **STANDARDIZED TEST PRACTICE**

Which value of x makes $7x - 10 = 9x$ true?

- (A) -5 (B) -4 (C) 4 (D) 5

3-3

The Real Number System

What You'll LEARN

Identify and classify numbers in the real number system.

NEW Vocabulary

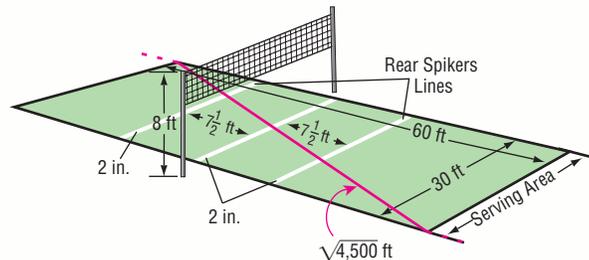
irrational number
real number

REVIEW Vocabulary

rational number: any number that can be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$
(Lesson 2-1)

WHEN am I ever going to use this?

SPORTS Most sports have rules for the size of the field or court where the sport is played. A diagram of a volleyball court is shown.



1. The length of the court is 60 feet. Is this number a whole number? Is it a rational number? Explain.
2. The distance from the net to the rear spikers line is $7\frac{1}{2}$ feet. Is this number a whole number? Is it a rational number? Explain.
3. The diagonal across the court is $\sqrt{4,500}$ feet. Can this square root be written as a whole number? a rational number?

Use a calculator to find $\sqrt{4,500}$. $\sqrt{4,500} \approx 67.08203932 \dots$

Although the decimal value of $\sqrt{4,500}$ continues on and on, it does not repeat. Since the decimal does not terminate or repeat, $\sqrt{4,500}$ is *not* a rational number. Numbers that are not rational are called **irrational numbers**. The square root of any number that is not a perfect square is irrational.

Noteables™

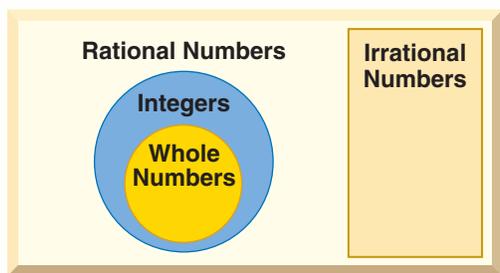
Key Concept: Irrational Numbers

Words An irrational number is a number that cannot be expressed as $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

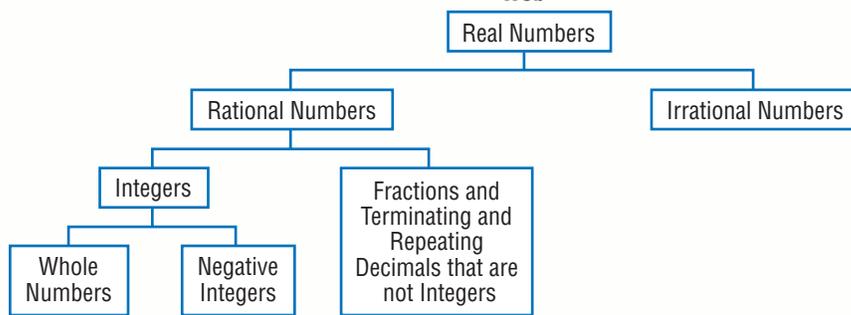
Symbols $\sqrt{2} \approx 1.414213562 \dots$ $\sqrt{3} \approx 1.732050808 \dots$

The set of rational numbers and the set of irrational numbers together make up the set of **real numbers**. Study the diagrams below.

Venn Diagram
Real Numbers



Web



STUDY TIP

Classifying Numbers Always simplify numbers before classifying them.

EXAMPLES

Classify Numbers

Name all sets of numbers to which each real number belongs.

- 1 $0.252525 \dots$ The decimal ends in a repeating pattern. It is a rational number because it is equivalent to $\frac{25}{99}$.
- 2 $\sqrt{36}$ Since $\sqrt{36} = 6$, it is a whole number, an integer, and a rational number.
- 3 $-\sqrt{7}$ $-\sqrt{7} \approx -2.645751311 \dots$ Since the decimal does not terminate or repeat, it is an irrational number.

Real numbers follow the number properties that are true for whole numbers, integers, and rational numbers.

Concept Summary

Real Number Properties

Property	Arithmetic	Algebra
Commutative	$3.2 + 2.5 = 2.5 + 3.2$ $5.1 \cdot 2.8 = 2.8 \cdot 5.1$	$a + b = b + a$ $a \cdot b = b \cdot a$
Associative	$(2 + 1) + 5 = 2 + (1 + 5)$ $(3 \cdot 4) \cdot 6 = 3 \cdot (4 \cdot 6)$	$(a + b) + c = a + (b + c)$ $(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Distributive	$2(3 + 5) = 2 \cdot 3 + 2 \cdot 5$	$a(b + c) = a \cdot b + a \cdot c$
Identity	$\sqrt{8} + 0 = \sqrt{8}$ $\sqrt{7} \cdot 1 = \sqrt{7}$	$a + 0 = a$ $a \cdot 1 = a$
Additive Inverse	$4 + (-4) = 0$	$a + (-a) = 0$
Multiplicative Inverse	$\frac{2}{3} \cdot \frac{3}{2} = 1$	$\frac{a}{b} \cdot \frac{b}{a} = 1$, where $a, b \neq 0$

The graph of all real numbers is the entire number line without any “holes.”

EXAMPLE

Graph Real Numbers

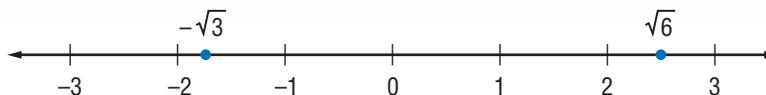
- 4 Estimate $\sqrt{6}$ and $-\sqrt{3}$ to the nearest tenth. Then graph $\sqrt{6}$ and $-\sqrt{3}$ on a number line.

Use a calculator to determine the approximate decimal values.

$$\sqrt{6} \approx 2.449489743 \dots$$

$$-\sqrt{3} \approx -1.7320508080 \dots$$

$\sqrt{6} \approx 2.4$ and $-\sqrt{3} \approx -1.7$. Locate these points on the number line.



- Your Turn Estimate each square root to the nearest tenth. Then graph the square root on a number line.

a. $\sqrt{5}$

b. $-\sqrt{7}$

c. $\sqrt{22}$

STUDY TIP

Mental Math

Remember that a negative number is always less than a positive number. Therefore, you can determine that $-\sqrt{3}$ is less than 1.7 without computation.

To compare real numbers, you can use a number line.

EXAMPLES

Compare Real Numbers

Replace each \bullet with $<$, $>$, or $=$ to make a true sentence.

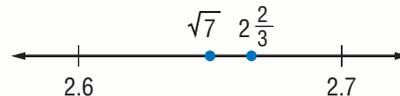
5 $\sqrt{7} \bullet 2\frac{2}{3}$

Write each number as a decimal.

$$\sqrt{7} \approx 2.645751311 \dots$$

$$2\frac{2}{3} = 2.666666666 \dots$$

Since 2.645751311... is less than 2.666666666..., $\sqrt{7} < 2\frac{2}{3}$.

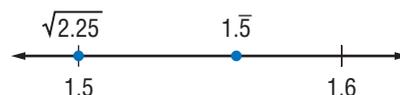


6 $1.\bar{5} \bullet \sqrt{2.25}$

Write $\sqrt{2.25}$ as a decimal.

$$\sqrt{2.25} = 1.5$$

Since $1.\bar{5}$ is greater than 1.5, $1.\bar{5} > \sqrt{2.25}$.



7 **Your Turn** Replace each \bullet with $<$, $>$, or $=$ to make a true sentence.

d. $\sqrt{11} \bullet 3\frac{1}{3}$

e. $\sqrt{17} \bullet 4.03$

f. $\sqrt{6.25} \bullet 2\frac{1}{2}$

REAL-LIFE CAREERS

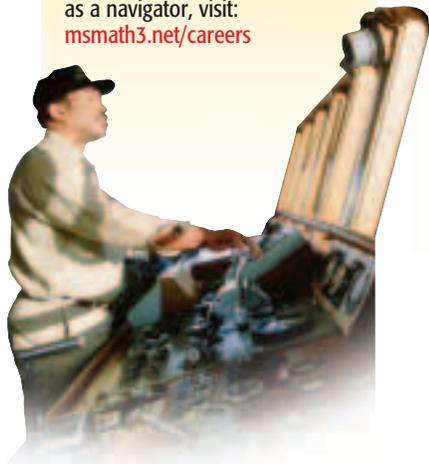
How Does a Navigator Use Math?

Navigators use math to calculate the course of a ship. They sometimes use lighthouses as landmarks in their navigating.



Research

For information about a career as a navigator, visit: msmath3.net/careers



EXAMPLE

Use Real Numbers

7 **LIGHTHOUSES** On a clear day, the number of miles a person can see to the horizon is about 1.23 times the square root of his or her distance from the ground, in feet. Suppose Domingo is at the top of the lighthouse at Cape Hatteras and Jewel is at the top of the lighthouse at Cape Lookout. How much farther can Domingo see than Jewel?

Use a calculator to approximate the distance each person can see.

$$\text{Domingo: } 1.23\sqrt{196} = 17.22 \quad \text{Jewel: } 1.23\sqrt{169} = 15.99$$

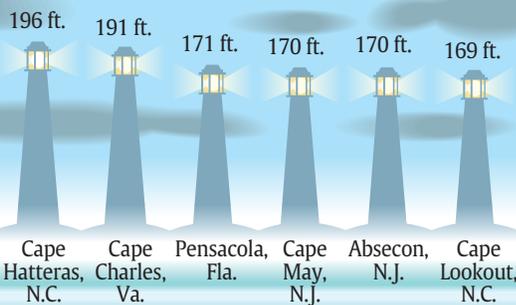
Domingo can see about $17.22 - 15.99$ or 1.23 miles farther than Jewel.



USA TODAY Snapshots®

Tallest lighthouses

The U.S. Lighthouse Society announced last month it will convert the former U.S. Lighthouse Service headquarters on New York's Staten Island into a national lighthouse museum. Tallest of the estimated 850 U.S. lighthouses:



Source: U.S. Lighthouse Society, San Francisco

By Anne R. Carey and Sam Ward, USA TODAY



msmath3.net/extra_examples

Skill and Concept Check

1. Give a counterexample for the statement *all square roots are irrational numbers*.
2. **OPEN ENDED** Write an irrational number which would be graphed between 7 and 8 on the number line.
3. **Which One Doesn't Belong?** Identify the number that is not the same type as the other three. Explain your reasoning.

$\sqrt{7}$

$\sqrt{11}$

$\sqrt{25}$

$\sqrt{35}$

GUIDED PRACTICE

Name all sets of numbers to which each real number belongs.

4. 0.050505... 5. $-\sqrt{100}$ 6. $\sqrt{17}$ 7. $-3\frac{1}{4}$

Estimate each square root to the nearest tenth. Then graph the square root on a number line.

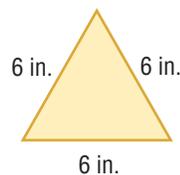
8. $\sqrt{2}$ 9. $-\sqrt{18}$ 10. $-\sqrt{30}$ 11. $\sqrt{95}$

Replace each \bullet with $<$, $>$, or $=$ to make a true sentence.

12. $\sqrt{15} \bullet 3.5$ 13. $\sqrt{2.25} \bullet 1\frac{1}{2}$ 14. $2.\overline{21} \bullet \sqrt{5.2}$

15. Order $5.\overline{5}$, $\sqrt{30}$, $5\frac{1}{2}$, and 5.56 from least to greatest.

16. **GEOMETRY** The area of a triangle with all three sides the same length is $\frac{s^2\sqrt{3}}{4}$, where s is the length of a side. Find the area of the triangle.



Practice and Applications

Name all sets of numbers to which each real number belongs.

17. 14 18. $\frac{2}{3}$ 19. $-\sqrt{16}$ 20. $-\sqrt{20}$
 21. 4.83 22. $7.\overline{2}$ 23. $-\sqrt{90}$ 24. $\frac{12}{4}$
 25. $-0.\overline{182}$ 26. -13 27. $5\frac{3}{8}$ 28. -108.6

29. Are integers *always*, *sometimes*, or *never* rational numbers? Explain.
 30. Are rational numbers *always*, *sometimes*, or *never* integers? Explain.

Estimate each square root to the nearest tenth. Then graph the square root on a number line.

31. $\sqrt{6}$ 32. $\sqrt{8}$ 33. $-\sqrt{22}$ 34. $-\sqrt{27}$
 35. $\sqrt{50}$ 36. $-\sqrt{48}$ 37. $-\sqrt{105}$ 38. $\sqrt{150}$

HOMework HELP

For Exercises	See Examples
17-30	1-3
31-38	4
39-48	5, 6
49-50	7

Extra Practice
See pages 622, 650.

Replace each \bullet with $<$, $>$, or $=$ to make a true sentence.

39. $\sqrt{10} \bullet 3.2$

40. $\sqrt{12} \bullet 3.5$

41. $6\frac{1}{3} \bullet \sqrt{40}$

42. $2\frac{2}{5} \bullet \sqrt{5.76}$

43. $5\frac{1}{6} \bullet 5.1\bar{6}$

44. $\sqrt{6.2} \bullet 2.\bar{4}$

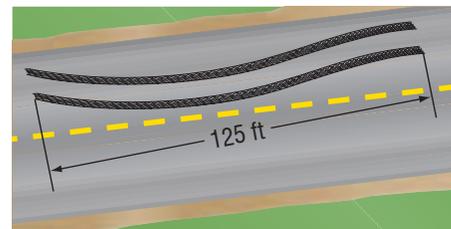
45. Order $\sqrt{5}$, $\sqrt{3}$, 2.25, and $2.\bar{2}$ from least to greatest.

46. Order 3.01, $3.\bar{1}$, $3.\overline{01}$, and $\sqrt{9}$ from least to greatest.

47. Order -4.1 , $\sqrt{17}$, $-4.\bar{1}$, and 4.01 from greatest to least.

48. Order $-\sqrt{5}$, $\sqrt{6}$, -2.5 , and 2.5 from greatest to least.

49. **LAW ENFORCEMENT** Traffic police can use the formula $s = 5.5\sqrt{0.75d}$ to estimate the speed of a vehicle before braking. In this formula, s is the speed of the vehicle in miles per hour, and d is the length of the skid marks in feet. How fast was the vehicle going for the skid marks at the right?



50. **WEATHER** Meteorologists use the formula $t^2 = \frac{d^3}{216}$ to estimate the amount of time that a thunderstorm will last. In this formula, t is the time in hours, and d is the distance across the storm in miles. How long will a thunderstorm that is 8.4 miles wide last?

51. **CRITICAL THINKING** Tell whether the following statement is *always*, *sometimes*, or *never* true.

The product of a rational number and an irrational number is an irrational number.

Spiral Review with Standardized Test Practice

52. **MULTIPLE CHOICE** To which set of numbers does $-\sqrt{49}$ *not* belong?

- (A) whole (B) rational (C) integers (D) real

53. **SHORT RESPONSE** The area of a square playground is 361 square feet. What is the perimeter of the playground?

54. Order 7, $\sqrt{53}$, $\sqrt{32}$, and 6 from least to greatest. (Lesson 3-2)

Solve each equation. (Lesson 3-1)

55. $t^2 = 25$

56. $y^2 = \frac{1}{49}$

57. $0.64 = a^2$

58. **ARCHAEOLOGY** Stone tools found in Ethiopia are estimated to be 2.5 million years old. That is about 700,000 years older than similar tools found in Tanzania. Write and solve an addition equation to find the age of the tools found in Tanzania. (Lesson 1-8)

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL Evaluate each expression. (Lesson 1-2)

59. $3^2 + 5^2$

60. $6^2 + 4^2$

61. $9^2 + 11^2$

62. $4^2 + 7^2$



Mid-Chapter Practice Test

Vocabulary and Concepts

- Graph $\sqrt{50}$ on a number line. (Lesson 3-2)
- Write an irrational number that would be graphed between 11 and 12 on a number line. (Lesson 3-3)
- OPEN ENDED** Give an example of a number that is an integer but not a whole number. (Lesson 3-3)

Skills and Applications

Find each square root. (Lesson 3-1)

- | | | |
|------------------|---------------------------|------------------|
| 4. $\sqrt{1}$ | 5. $-\sqrt{81}$ | 6. $\sqrt{36}$ |
| 7. $-\sqrt{121}$ | 8. $-\sqrt{\frac{1}{25}}$ | 9. $\sqrt{0.09}$ |

10. **GEOMETRY** What is the length of a side of the square? (Lesson 3-1)

11. **ALGEBRA** Estimate the solution of $x^2 = 50$ to the nearest integer. (Lesson 3-2)

Area =
225 square
meters

Estimate to the nearest whole number. (Lesson 3-2)

- | | | |
|-----------------|-----------------|------------------|
| 12. $\sqrt{90}$ | 13. $\sqrt{28}$ | 14. $\sqrt{226}$ |
| 15. $\sqrt{17}$ | 16. $\sqrt{21}$ | 17. $\sqrt{75}$ |

Name all sets of numbers to which each real number belongs. (Lesson 3-3)

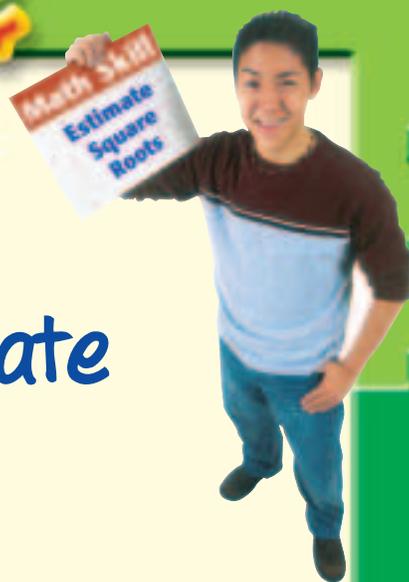
- | | | |
|-------------------|-----------------|------------------|
| 18. $\frac{2}{3}$ | 19. $\sqrt{25}$ | 20. $-\sqrt{15}$ |
| 21. $\sqrt{3}$ | 22. 10 | 23. $-\sqrt{4}$ |

Standardized Test Practice

24. **MULTIPLE CHOICE** The area of a square checkerboard is 529 square centimeters. How long is each side of the checkerboard? (Lesson 3-1)
- (A) 21 cm (B) 22 cm
(C) 23 cm (D) 24 cm
25. **MULTIPLE CHOICE** To which set of numbers does $\sqrt{\frac{144}{36}}$ not belong? (Lesson 3-3)
- (F) integers (G) rationals
(H) wholes (I) irrationals

The Game Zone

A Place To Practice Your Math Skills



Estimate and Eliminate

● GET READY!

Players: four

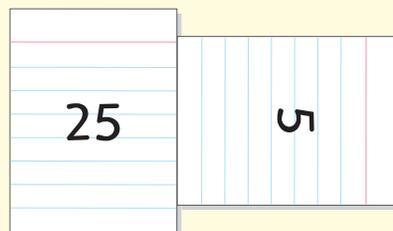
Materials: 40 index cards, 4 markers

● GET SET!

- Each player is given 10 index cards.
- Player 1 writes one of each of the whole numbers 1 to 10 on his or her cards. Player 2 writes the square of one of each of the whole numbers 1 to 10. Player 3 writes a different whole number between 11 and 50, that is not a perfect square. Player 4 writes a different whole number between 51 and 99, that is not a perfect square.

● GO!

- Mix all 40 cards together. The dealer deals all of the cards.
- In turn, moving clockwise, each player lays down any pair(s) of a perfect square and its square root in his or her hand. The two cards should be laid down as shown at the right. If a player has no perfect square and square root pair, he or she skips a turn.
- After the first round, any player, during his or her turn may:
 - (1) lay down a perfect square and square root pair, or
 - (2) cover a card that is already on the table. The new card should form a square and *estimated* square root pair with the card next to it. A player makes as many plays as possible during his or her turn.
- After each round, each player passes one card left.
- **Who Wins?** The first person without any cards is the winner.

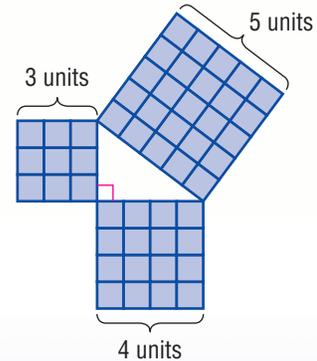


The Pythagorean Theorem

HANDS-ON Mini Lab

Materials

- grid paper



What You'll LEARN

Use the Pythagorean Theorem.

NEW Vocabulary

right triangle
legs
hypotenuse
Pythagorean Theorem
converse

Link to READING

Everyday Meaning of Leg: limb used to support the body

Work with a partner.

Three squares with sides 3, 4, and 5 units are used to form the right triangle shown.

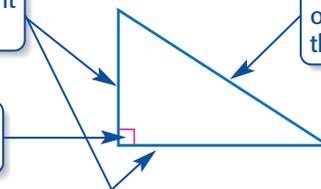
1. Find the area of each square.
2. How are the squares of the sides related to the areas of the squares?
3. Find the sum of the areas of the two smaller squares. How does the sum compare to the area of the larger square?
4. Use grid paper to cut out three squares with sides 5, 12, and 13 units. Form a right triangle with these squares. Compare the sum of the areas of the two smaller squares with the area of the larger square.

A **right triangle** is a triangle with one right angle. A right angle is an angle with a measure of 90° .

The sides that form the right angle are called **legs**.

The **hypotenuse** is the side opposite the right angle. It is the longest side of the triangle.

The symbol \square indicates a right angle.



The **Pythagorean Theorem** describes the relationship between the lengths of the legs and the hypotenuse for *any* right triangle.

Noteables™

Key Concept: Pythagorean Theorem

Words In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

Symbols

Arithmetic

$$5^2 = 3^2 + 4^2$$

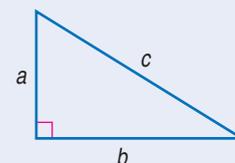
$$25 = 9 + 16$$

$$25 = 25$$

Algebra

$$c^2 = a^2 + b^2$$

Model



You can use the Pythagorean Theorem to find the length of a side of a right triangle.

REAL-LIFE MATH

KITES Some kites need as little as a 3-mile-per-hour breeze to fly. Others need a wind in excess of 10 miles per hour.

Source: World Book

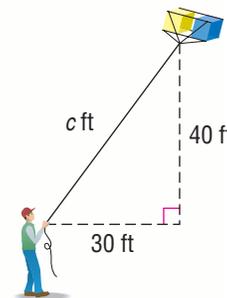


EXAMPLE Find the Length of the Hypotenuse

1 KITES Find the length of the kite string.

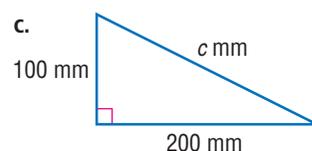
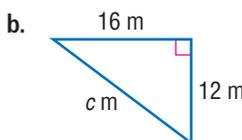
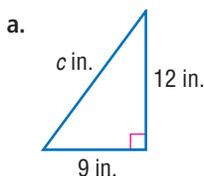
The kite string forms the hypotenuse of a right triangle. The vertical and horizontal distances form the legs.

$$\begin{aligned} c^2 &= a^2 + b^2 && \text{Pythagorean Theorem} \\ c^2 &= 30^2 + 40^2 && \text{Replace } a \text{ with } 30 \text{ and } b \text{ with } 40. \\ c^2 &= 900 + 1,600 && \text{Evaluate } 30^2 \text{ and } 40^2. \\ c^2 &= 2,500 && \text{Add } 900 \text{ and } 1,600. \\ \sqrt{c^2} &= \sqrt{2,500} && \text{Take the square root of each side.} \\ c &= 50 \text{ or } -50 && \text{Simplify.} \end{aligned}$$



The equation has two solutions, 50 and -50 . However, the length of the kite string must be positive. So, the kite string is 50 feet long.

Your Turn Find the length of each hypotenuse. Round to the nearest tenth if necessary.



EXAMPLE Find the Length of a Leg

2 The hypotenuse of a right triangle is 20 centimeters long and one of its legs is 17 centimeters. Find the length of the other leg.

$$\begin{aligned} c^2 &= a^2 + b^2 && \text{Pythagorean Theorem} \\ 20^2 &= a^2 + 17^2 && \text{Replace } c \text{ with } 20 \text{ and } b \text{ with } 17. \\ 400 &= a^2 + 289 && \text{Evaluate } 20^2 \text{ and } 17^2. \\ 400 - 289 &= a^2 + 289 - 289 && \text{Subtract } 289 \text{ from each side.} \\ 111 &= a^2 && \text{Simplify.} \\ \sqrt{111} &= \sqrt{a^2} && \text{Take the square root of each side.} \\ 10.5 &\approx a && \text{Use a calculator.} \end{aligned}$$

The length of the other leg is about 10.5 centimeters.

Your Turn Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

- d. b , 9 ft; c , 12 ft e. a , 3 m; c , 8 m f. a , 15 in.; b , 18 in.

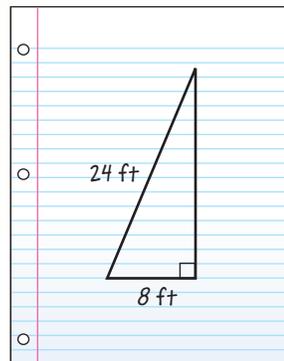


EXAMPLE Use the Pythagorean Theorem

3 MULTIPLE-CHOICE TEST ITEM For safety reasons, the base of a 24-foot ladder should be at least 8 feet from the wall. How high can a 24-foot ladder safely reach?

- (A) about 16 feet (B) about 22.6 feet
(C) about 25.3 feet (D) about 512 feet

Read the Test Item You know the length of the ladder and the distance from the base of the ladder to the side of the house. Make a drawing of the situation including the right triangle.



Solve the Test Item Use the Pythagorean Theorem.

$$c^2 = a^2 + b^2 \quad \text{Pythagorean Theorem}$$

$$24^2 = a^2 + 8^2 \quad \text{Replace } c \text{ with } 24 \text{ and } b \text{ with } 8.$$

$$576 = a^2 + 64 \quad \text{Evaluate } 24^2 \text{ and } 8^2.$$

$$576 - 64 = a^2 + 64 - 64 \quad \text{Subtract } 64 \text{ from each side.}$$

$$512 = a^2 \quad \text{Simplify.}$$

$$\sqrt{512} = \sqrt{a^2} \quad \text{Take the square root of each side.}$$

$$22.6 \approx a \quad \text{Use a calculator.}$$

The ladder can safely reach a height of 22.6 feet. The answer is B.

Test-Taking Tip

Draw a Picture When solving a problem, it is sometimes helpful to draw a picture to represent the situation.

If you reverse the parts of the Pythagorean Theorem, you have formed its **converse**. The converse of the Pythagorean Theorem is also true.

Noteables™ Key Concept: Converse of Pythagorean Theorem

If the sides of a triangle have lengths a , b , and c units such that $c^2 = a^2 + b^2$, then the triangle is a right triangle.

EXAMPLE Identify a Right Triangle

4 The measures of three sides of a triangle are 15 inches, 8 inches, and 17 inches. Determine whether the triangle is a right triangle.

$$c^2 = a^2 + b^2 \quad \text{Pythagorean Theorem}$$

$$17^2 \stackrel{?}{=} 15^2 + 8^2 \quad c = 17, a = 15, b = 8$$

$$289 \stackrel{?}{=} 225 + 64 \quad \text{Evaluate } 17^2, 15^2, \text{ and } 8^2.$$

$$289 = 289 \quad \checkmark \quad \text{Simplify.}$$

The triangle is a right triangle.

Your Turn Determine whether each triangle with sides of given lengths is a right triangle.

- g. 18 mi, 24 mi, 30 mi h. 4 ft, 7 ft, 5 ft

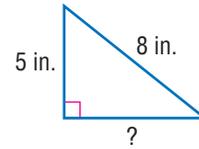
STUDY TIP

Assigning Variables

Remember that the longest side of a right triangle is the hypotenuse. Therefore, c represents the length of the longest side.

Skill and Concept Check

1. Draw a right triangle and label all the parts.
2. **OPEN ENDED** State three measures that could be the side measures of a right triangle.
3. **FIND THE ERROR** Catalina and Morgan are writing an equation to find the length of the third side of the triangle. Who is correct? Explain.

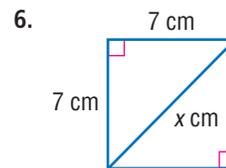
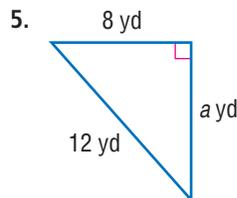
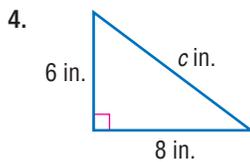


Catalina
 $c^2 = 5^2 + 8^2$

Morgan
 $8^2 = a^2 + 5^2$

GUIDED PRACTICE

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.



7. $a, 5$ ft; $c, 6$ ft

8. $a, 9$ m; $b, 7$ m

9. $b, 4$ yd; $c, 10$ yd

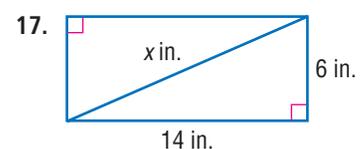
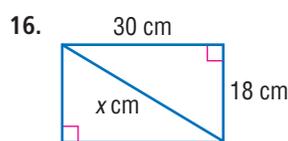
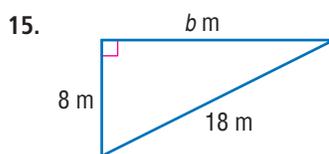
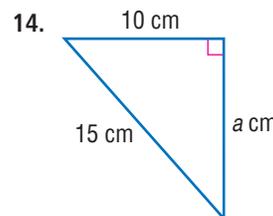
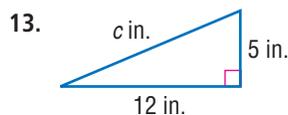
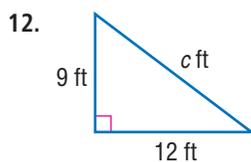
Determine whether each triangle with sides of given lengths is a right triangle.

10. 5 in., 10 in., 12 in.

11. 9 m, 40 m, 41 m

Practice and Applications

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.



18. $b, 99$ mm; $c, 101$ mm

19. $a, 48$ yd; $b, 55$ yd

20. $a, 17$ ft; $c, 20$ ft

21. $a, 23$ in.; $b, 18$ in.

22. $b, 4.5$ m; $c, 9.4$ m

23. $b, 5.1$ m; $c, 12.3$ m

HOMEWORK HELP

For Exercises	See Examples
12–25, 32	1–3
26–31, 34	4

Extra Practice
See pages 623, 650.



24. The hypotenuse of a right triangle is 12 inches, and one of its legs is 7 inches. Find the length of the other leg.
25. If one leg of a right triangle is 8 feet and its hypotenuse is 14 feet, how long is the other leg?

Determine whether each triangle with sides of given lengths is a right triangle.

26. 28 yd, 195 yd, 197 yd 27. 30 cm, 122 cm, 125 cm 28. 24 m, 143 m, 145 m
 29. 135 in., 140 in., 175 in. 30. 56 ft, 65 ft, 16 ft 31. 44 cm, 70 cm, 55 cm

32. **GEOGRAPHY** Wyoming's rectangular shape is about 275 miles by 365 miles. Find the length of the diagonal of the state of Wyoming.

33. **RESEARCH** Use the Internet or other resource to find the measurements of another state. Then calculate the length of a diagonal of the state.

34. **TRAVEL** The Research Triangle in North Carolina is formed by Raleigh, Durham, and Chapel Hill. Is this triangle a right triangle? Explain.



35. **CRITICAL THINKING** About 2000 B.C., Egyptian engineers discovered a way to make a right triangle using a rope with 12 evenly spaced knots tied in it. They attached one end of the rope to a stake in the ground. At what knot locations should the other two stakes be placed in order to form a right triangle? Draw a diagram.

Spiral Review with Standardized Test Practice

36. **MULTIPLE CHOICE** A hiker walked 22 miles north and then walked 17 miles west. How far is the hiker from the starting point?

- (A) 374 mi (B) 112.6 mi (C) 39 mi (D) 27.8 mi

37. **SHORT RESPONSE** What is the perimeter of a right triangle if the lengths of the legs are 10 inches and 24 inches?

Replace each \bullet with $<$, $>$, or $=$ to make each a true sentence. (Lesson 3-3)

38. $\sqrt{12} \bullet 3.5$ 39. $\sqrt{41} \bullet 6.4$ 40. $5.\bar{6} \bullet \frac{17}{3}$ 41. $\sqrt{55} \bullet 7.\bar{4}$

42. **ALGEBRA** Estimate the solution of $x^2 = 77$ to the nearest integer. (Lesson 3-2)

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL Solve each equation. Check your solution. (Lesson 1-8)

43. $57 = x + 24$ 44. $82 = 54 + y$ 45. $71 = 35 + z$ 46. $64 = a + 27$

3-5

Using the Pythagorean Theorem

What You'll LEARN

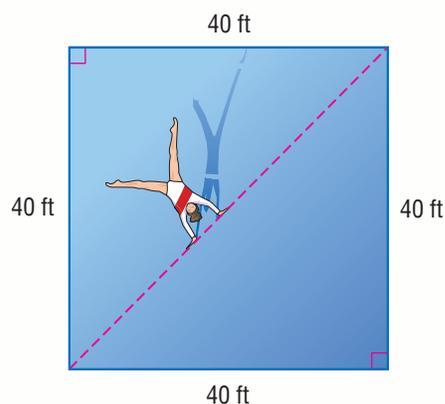
Solve problems using the Pythagorean Theorem.

NEW Vocabulary

Pythagorean triple

WHEN am I ever going to use this?

GYMNASTICS In the floor exercises of women's gymnastics, athletes cross the diagonal of the mat flipping and twisting as they go. It is important that the gymnast does not step off the mat.



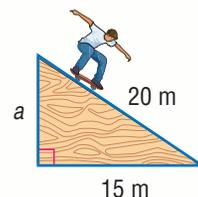
1. What type of triangle is formed by the sides of the mat and the diagonal?
2. Write an equation that can be used to find the length of the diagonal.

The Pythagorean Theorem can be used to solve a variety of problems.

EXAMPLE Use the Pythagorean Theorem

1 SKATEBOARDING Find the height of the skateboard ramp.

Notice the problem involves a right triangle. Use the Pythagorean Theorem.



Words	The square of the hypotenuse	equals	the sum of the squares of the legs.
Variables	c^2	=	$a^2 + b^2$
Equation	20^2	=	$a^2 + 15^2$

$$20^2 = a^2 + 15^2$$

Write the equation.

$$400 = a^2 + 225$$

Evaluate 20^2 and 15^2 .

$$400 - 225 = a^2 + 225 - 225$$

Subtract 225 from each side.

$$175 = a^2$$

Simplify.

$$\sqrt{175} = \sqrt{a^2}$$

Take the square root of each side.

$$13.2 \approx a$$

Simplify.

The height of the ramp is about 13.2 meters.



You know that a triangle with sides 3, 4, and 5 units is a right triangle because these numbers satisfy the Pythagorean Theorem. Such whole numbers are called **Pythagorean triples**. By using multiples of a Pythagorean triple, you can create additional triples.

EXAMPLE Write Pythagorean Triples

- 2** Multiply the triple 3-4-5 by the numbers 2, 3, 4, and 10 to find more Pythagorean triples.

You can organize your answers in a table. Multiply each Pythagorean triple entry by the same number and then check the Pythagorean relationship.

	a	b	c	Check: $c^2 = a^2 + b^2$
original	3	4	5	$25 = 9 + 16$ ✓
$\times 2$	6	8	10	$100 = 36 + 64$ ✓
$\times 3$	9	12	15	$225 = 81 + 144$ ✓
$\times 4$	12	16	20	$400 = 144 + 256$ ✓
$\times 10$	30	40	50	$2,500 = 900 + 1,600$ ✓

Skill and Concept Check

- Writing Math** Explain why you can use any two sides of a right triangle to find the third side.
- OPEN ENDED** Write a problem that can be solved by using the Pythagorean Theorem. Then solve the problem.
- Which One Doesn't Belong?** Identify the set of numbers that are not Pythagorean triples. Explain your reasoning.

5-12-13

10-24-26

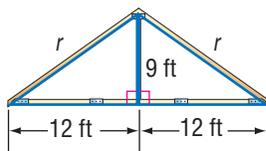
5-7-9

8-15-17

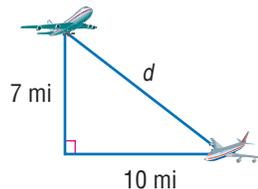
GUIDED PRACTICE

Write an equation that can be used to answer the question. Then solve. Round to the nearest tenth if necessary.

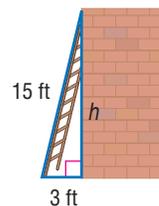
4. How long is each rafter?



5. How far apart are the planes?



6. How high does the ladder reach?



7. **GEOMETRY** An *isosceles* right triangle is a right triangle in which both legs are equal in length. If the leg of an isosceles triangle is 4 inches long, what is the length of the hypotenuse?

Practice and Applications

HOMWORK HELP

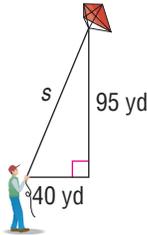
For Exercises 8–19 See Examples

1, 2

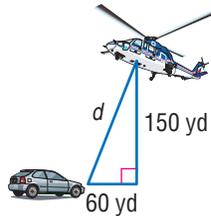
Extra Practice
See pages 623, 650.

Write an equation that can be used to answer the question. Then solve. Round to the nearest tenth if necessary.

8. How long is the kite string?



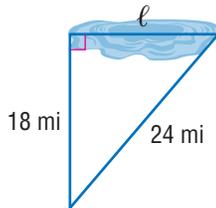
9. How far is the helicopter from the car?



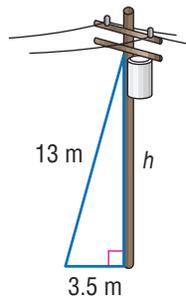
10. How high is the ski ramp?



11. How long is the lake?



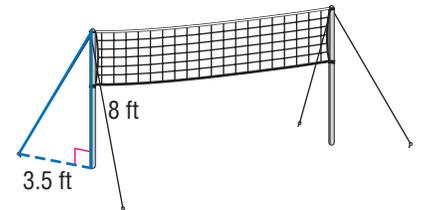
12. How high is the wire attached to the pole?



13. How high is the wheel chair ramp?



14. **VOLLEYBALL** Two ropes and two stakes are needed to support each pole holding the volleyball net. Find the length of each rope.

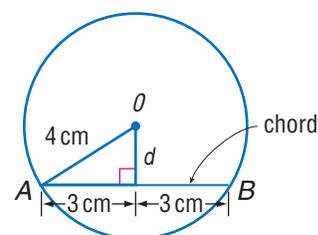


15. **ENTERTAINMENT** Connor loves to watch movies in the letterbox format on his television. He wants to buy a new television with a screen that is at least 25 inches by 13.6 inches. What diagonal size television meets Connor's requirements?

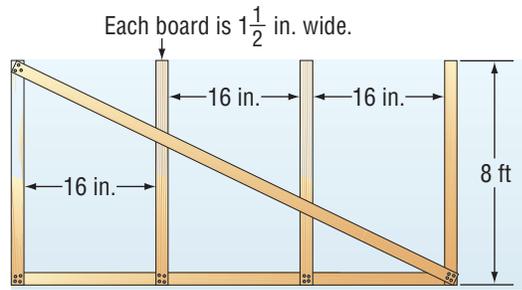
16. **GEOGRAPHY** Suppose Flint, Ann Arbor, and Kalamazoo, Michigan, form a right triangle. What is the distance from Kalamazoo to Ann Arbor?



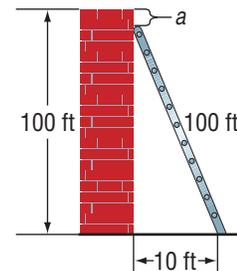
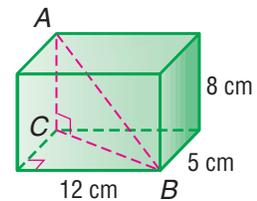
17. **GEOMETRY** A line segment with endpoints on a circle is called a *chord*. Find the distance d from the center of the circle O to the chord \overline{AB} in the circle below.



18. **MULTI STEP** Home builders add corner bracing to give strength to a house frame. How long will the brace need to be for the frame below?



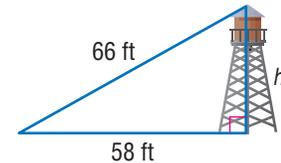
19. **GEOMETRY** Find the length of the diagonal \overline{AB} in the rectangular prism at the right. (*Hint*: First find the length of \overline{BC} .)
20. **MODELING** Measure the dimensions of a shoebox and use the dimensions to calculate the length of the diagonal of the box. Then use a piece of string and a ruler to check your calculation.
21. **CRITICAL THINKING** Suppose a ladder 100 feet long is placed against a vertical wall 100 feet high. How far would the top of the ladder move down the wall by pulling out the bottom of the ladder 10 feet?



Spiral Review with Standardized Test Practice

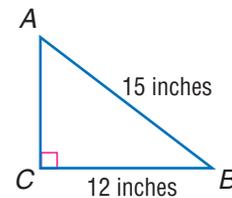
22. **MULTIPLE CHOICE** What is the height of the tower?

- (A) 8 feet (B) 31.5 feet
(C) 49.9 feet (D) 992 feet



23. **MULTIPLE CHOICE** Triangle ABC is a right triangle. What is the perimeter of the triangle?

- (F) 3 in. (G) 9 in.
(H) 27 in. (I) 36 in.



24. **GEOMETRY** Determine whether a triangle with sides 20 inches, 48 inches, and 52 inches long is a right triangle. (Lesson 3-4)

25. Order $\sqrt{45}$, $6.\bar{6}$, 6.75 , and 6.7 from least to greatest. (Lesson 3-3)

Evaluate each expression. (Lesson 2-8)

26. 2^4 27. 3^3 28. $2^3 \cdot 3^2$ 29. $10^5 \cdot 4^2$

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL Graph each point on a coordinate plane. (Page 614)

30. $T(5, 2)$ 31. $A(-1, 3)$ 32. $M(-5, 0)$ 33. $D(-2, -4)$

What You'll LEARN

Graph irrational numbers.

Materials

- grid paper
- compass
- straightedge

Graphing Irrational Numbers

In Lesson 3-3, you found approximate locations for irrational numbers on a number line. You can accurately graph irrational numbers.

ACTIVITY

Work with a partner.

Graph $\sqrt{34}$ on a number line as accurately as possible.**STEP 1**

Find two numbers whose squares have a sum of 34.

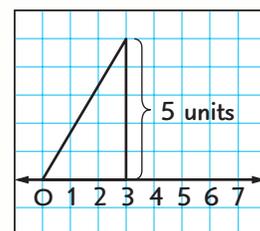
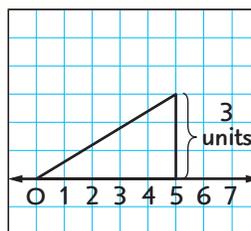
$$34 = 25 + 9$$

$$34 = 5^2 + 3^2$$

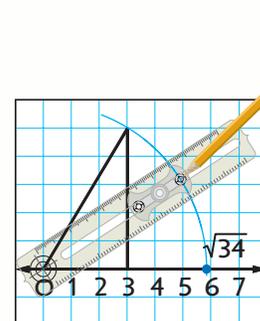
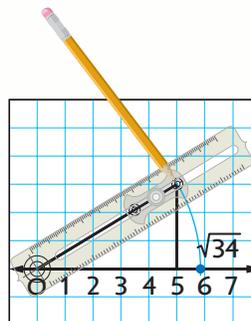
The hypotenuse of a triangle with legs that measure 5 and 3 units will measure $\sqrt{34}$ units.

STEP 2

Draw a number line on grid paper. Then draw a triangle whose legs measure 5 and 3 units.

**STEP 3**

Adjust your compass to the length of the hypotenuse. Place the compass at 0, draw an arc that intersects the number line. The point of intersection is the graph of $\sqrt{34}$.



Your Turn Accurately graph each irrational number.

a. $\sqrt{10}$

b. $\sqrt{13}$

c. $\sqrt{17}$

d. $\sqrt{8}$

Writing Math

1. **Explain** how you decide what lengths to make the legs of the right triangle when graphing an irrational number.
2. **Explain** how the graph of $\sqrt{2}$ can be used to graph $\sqrt{3}$.
3. **MAKE A CONJECTURE** Do you think you could graph the square root of any whole number? Explain.

Geometry: Distance on the Coordinate Plane

What You'll LEARN

Find the distance between points on the coordinate plane.

NEW Vocabulary

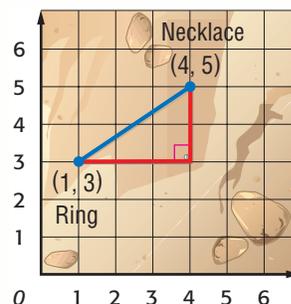
coordinate plane
origin
y-axis
x-axis
quadrants
ordered pair
x-coordinate
abscissa
y-coordinate
ordinate

REVIEW Vocabulary

integers: whole numbers and their opposites
(Lesson 1-3)

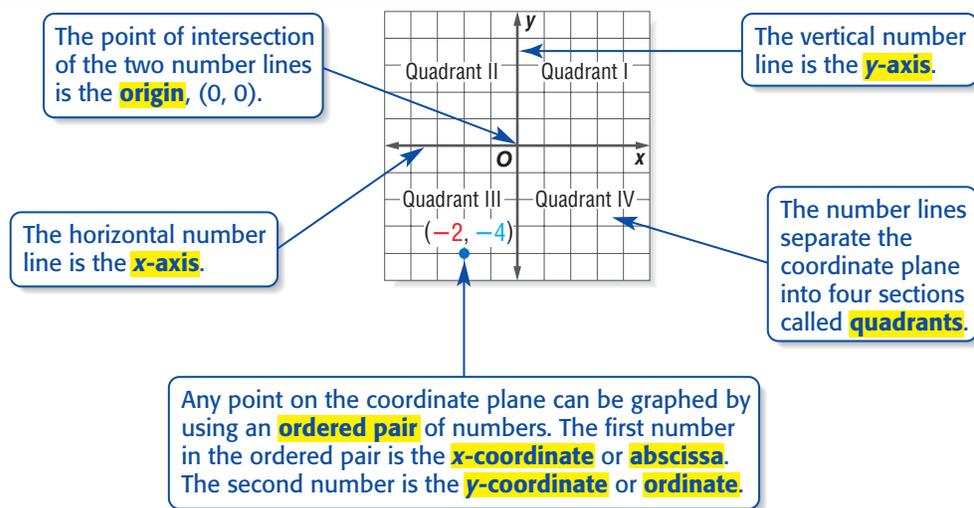
WHEN am I ever going to use this?

ARCHAEOLOGY Archaeologists keep careful records of the exact locations of objects found at digs. To accomplish this, they set up grids with string. Suppose a ring is found at $(1, 3)$ and a necklace is found at $(4, 5)$. The distance between the locations of these two objects is represented by the blue line.



1. What type of triangle is formed by the blue and red lines?
2. What is the length of the two red lines?
3. Write an equation you could use to determine the distance d between the locations where the ring and necklace were found.
4. How far apart were the ring and the necklace?

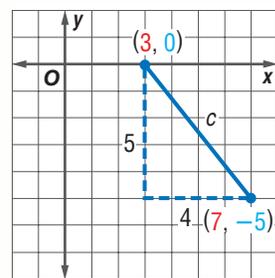
In mathematics, you can locate a point by using a coordinate system similar to the grid system used by archaeologists. A **coordinate plane** is formed by two number lines that form right angles and intersect at their zero points.



You can use the Pythagorean Theorem to find the distance between two points on the coordinate plane.

EXAMPLE**Find Distance on the Coordinate Plane**

- 1** Graph the ordered pairs $(3, 0)$ and $(7, -5)$. Then find the distance between the points.



Let c = the distance between the two points, $a = 4$, and $b = 5$.

$$c^2 = a^2 + b^2 \quad \text{Pythagorean Theorem}$$

$$c^2 = 4^2 + 5^2 \quad \text{Replace } a \text{ with 4 and } b \text{ with 5.}$$

$$c^2 = 16 + 25 \quad \text{Evaluate } 4^2 \text{ and } 5^2.$$

$$c^2 = 41 \quad \text{Add 16 and 25.}$$

$$\sqrt{c^2} = \sqrt{41} \quad \text{Take the square root of each side.}$$

$$c \approx 6.4 \quad \text{Simplify.}$$

The points are about 6.4 units apart.

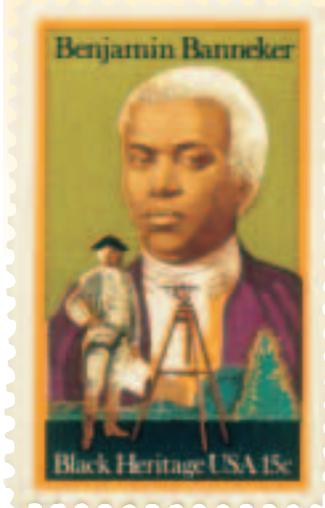
- Your Turn** Graph each pair of ordered pairs. Then find the distance between the points. Round to the nearest tenth if necessary.

- a. $(2, 0), (5, -4)$ b. $(1, 3), (-2, 4)$ c. $(-3, -4), (2, -1)$

REAL-LIFE MATH

TRAVEL Benjamin Banneker helped to survey and lay out Washington, D.C. He also made all the astronomical and tide calculations for the almanac he published.

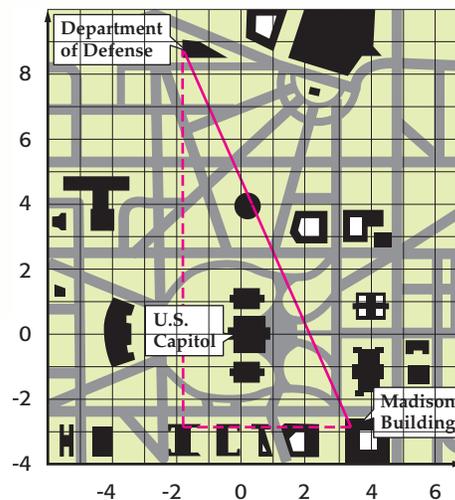
Source: *World Book*



You can use this technique to find distances on a map.

EXAMPLE**Find Distance on a Map**

- 2** **TRAVEL** The Yeager family is visiting Washington, D.C. A unit on the grid of their map shown at the right is 0.05 mile. Find the distance between the Department of Defense at $(-2, 9)$ and the Madison Building at $(3, -3)$.



Let c = the distance between the Department of Defense and the Madison Building. Then $a = 5$ and $b = 12$.

$$c^2 = a^2 + b^2 \quad \text{Pythagorean Theorem}$$

$$c^2 = 5^2 + 12^2 \quad \text{Replace } a \text{ with 5 and } b \text{ with 12.}$$

$$c^2 = 25 + 144 \quad \text{Evaluate } 5^2 \text{ and } 12^2.$$

$$c^2 = 169 \quad \text{Add 25 and 144.}$$

$$\sqrt{c^2} = \sqrt{169} \quad \text{Take the square root of each side.}$$

$$c = 13 \quad \text{Simplify.}$$

The distance between the Department of Defense and the Madison Building is 13 units on the map. Since each unit equals 0.05 mile, the distance between the two buildings is $0.05 \cdot 13$ or 0.65 mile.

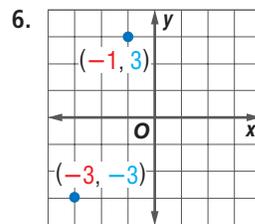
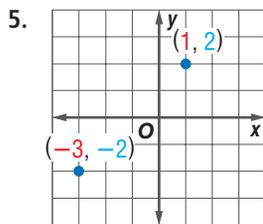
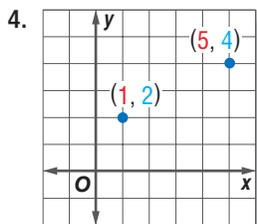


Skill and Concept Check

1. **Name** the theorem that is used to find the distance between two points on the coordinate plane.
2. **Draw** a triangle that you can use to find the distance between points at $(-3, 2)$ and $(-6, -4)$.
3. **OPEN ENDED** Give the coordinates of a line segment that is neither horizontal nor vertical and has a length of 5 units.

GUIDED PRACTICE

Find the distance between each pair of points whose coordinates are given. Round to the nearest tenth if necessary.



Graph each pair of ordered pairs. Then find the distance between the points. Round to the nearest tenth if necessary.

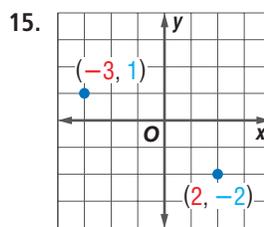
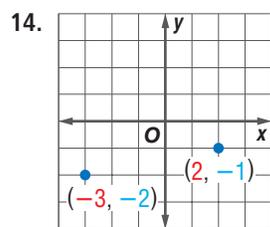
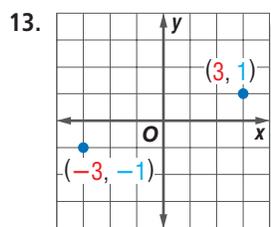
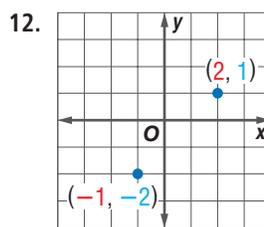
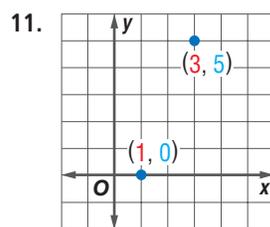
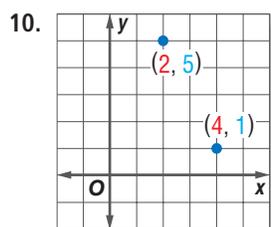
7. $(1, 5), (3, 1)$

8. $(-1, 0), (2, 7)$

9. $(-5, -2), (2, 3)$

Practice and Applications

Find the distance between each pair of points whose coordinates are given. Round to the nearest tenth if necessary.



Graph each pair of ordered pairs. Then find the distance between the points. Round to the nearest tenth if necessary.

16. $(4, 5), (2, 2)$

17. $(6, 2), (1, 0)$

18. $(-3, 4), (1, 3)$

19. $(-5, 1), (2, 4)$

20. $(2.5, -1), (-3.5, -5)$

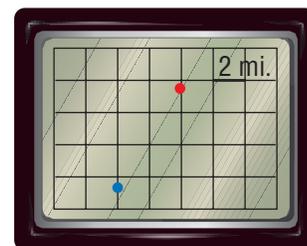
21. $(4, -2.3), (-1, -6.3)$

HOMEWORK HELP

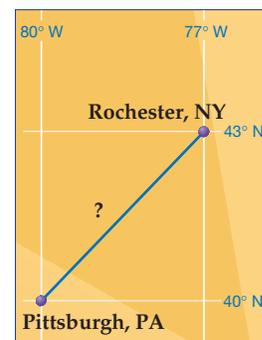
For Exercises	See Examples
10–21	1
22–23	2

Extra Practice
See pages 623, 650.

22. **TECHNOLOGY** A backpacker uses her GPS (Global Positioning System) receiver to find how much farther she needs to travel to get to her stopping point for the day. She is at the red dot on her GPS receiver screen and the blue dot shows her destination. How much farther does she need to travel?



23. **TRAVEL** Rochester, New York, has a longitude of 77° W and a latitude of 43° N. Pittsburgh, Pennsylvania, is located at 80° W and 40° N. At this longitude/latitude, each degree is about 53 miles. Find the distance between Rochester and Pittsburgh.



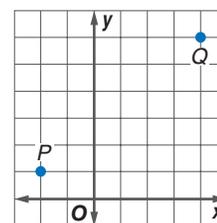
Data Update What is the distance between where you live and another place of your choice? Visit msmath3.net/data_update to find the longitude and latitude of each city.

24. **CRITICAL THINKING** The *midpoint* of a segment separates it into two parts of equal length. Find the midpoint of each horizontal or vertical line segment with coordinates of the endpoints given.
- a. $(5, 4), (5, 8)$ b. $(3, 2), (3, -4)$ c. $(-2, 5), (-2, -1)$ d. $(a, 5), (b, 5)$
25. **CRITICAL THINKING** Study your answers for Exercise 24. Write a rule for finding the midpoint of a horizontal or vertical line.

Spiral Review with Standardized Test Practice

26. **MULTIPLE CHOICE** Find the distance between P and Q .

- A 7.8 units B 8.5 units
 C 9.5 units D 9.0 units



27. **SHORT RESPONSE** Write an equation that can be used to find the distance between $M(-1, 3)$ and $N(3, 5)$.

28. **HIKING** Hunter hikes 3 miles south and then turns and hikes 7 miles east. How far is he from his starting point? (Lesson 3-5)

Find the missing side of each right triangle. Round to the nearest tenth.

(Lesson 3-4)

29. $a, 15$ cm; $b, 18$ cm 30. $b, 14$ in.; $c, 17$ in.

INTERDISCIPLINARY PROJECT

Bon Voyage!

Math and Geography It's time to complete your project. Use the information and data you have gathered about cruise packages and destination activities to prepare a video or brochure. Be sure to include a diagram and itinerary with your project.



msmath3.net/webquest



msmath3.net/self_check_quiz

Vocabulary and Concept Check

abscissa (p. 142)

converse (p. 134)

coordinate plane (p. 142)

hypotenuse (p. 132)

irrational number (p. 125)

legs (p. 132)

ordered pair (p. 142)

ordinate (p. 142)

origin (p. 142)

perfect square (p. 116)

principal square root (p. 117)

Pythagorean Theorem (p. 132)

Pythagorean triple (p. 138)

quadrants (p. 142)

radical sign (p. 116)

real number (p. 125)

right triangle (p. 132)

square root (p. 116)

x-axis (p. 142)

x-coordinate (p. 142)

y-axis (p. 142)

y-coordinate (p. 142)

State whether each sentence is *true* or *false*. If *false*, replace the underlined word(s) or number(s) to make a true sentence.

- An irrational number can be written as a fraction.
- The hypotenuse is the longest side of a right triangle.
- The set of numbers {3, 4, 5} is a Pythagorean triple.
- The number 11 is a perfect square.
- The horizontal axis is called the y -axis.
- In an ordered pair, the y -coordinate is the second number.
- The Pythagorean Theorem says that the sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse.
- The coordinates of the origin are (0, 1).

Lesson-by-Lesson Exercises and Examples

3-1

Square Roots (pp. 116–119)

Find each square root.

9. $\sqrt{81}$

10. $\sqrt{225}$

11. $\sqrt{64}$

12. $-\sqrt{100}$

13. $-\sqrt{\frac{4}{9}}$

14. $\sqrt{6.25}$

15. **FARMING** Pecan trees are planted in square patterns to take advantage of land space and for ease in harvesting. For 289 trees, how many rows should be planted and how many trees should be planted in each row?

Example 1 Find $\sqrt{36}$.

$\sqrt{36}$ indicates the *positive* square root of 36.

Since $6^2 = 36$, $\sqrt{36} = 6$.

Example 2 Find $-\sqrt{169}$.

$-\sqrt{169}$ indicates the *negative* square root of 169.

Since $(-13)(-13) = 169$, $-\sqrt{169} = -13$.

3-2 Estimating Square Roots (pp. 120–122)

Estimate to the nearest whole number.

- | | |
|-------------------|--------------------|
| 16. $\sqrt{32}$ | 17. $\sqrt{42}$ |
| 18. $\sqrt{230}$ | 19. $\sqrt{96}$ |
| 20. $\sqrt{150}$ | 21. $\sqrt{8}$ |
| 22. $\sqrt{50.1}$ | 23. $\sqrt{19.25}$ |

24. **ALGEBRA** Estimate the solution of $b^2 = 60$ to the nearest integer.

Example 3 Estimate $\sqrt{135}$ to the nearest whole number.

$$121 < 135 < 144 \quad \text{Write an inequality.}$$
$$11^2 < 135 < 12^2 \quad 121 = 11^2 \text{ and } 144 = 12^2$$
$$11 < \sqrt{135} < 12 \quad \text{Take the square root of each number.}$$

So, $\sqrt{135}$ is between 11 and 12. Since 135 is closer to 144 than to 121, the best whole number estimate is 12.

3-3 The Real Number System (pp. 125–129)

Name all sets of numbers to which each real number belongs.

- | | |
|------------------|-----------------|
| 25. $-\sqrt{19}$ | 26. $0.\bar{3}$ |
| 27. 7.43 | 28. -12 |
| 29. $\sqrt{32}$ | 30. 101 |

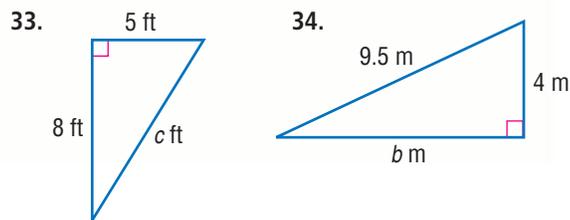
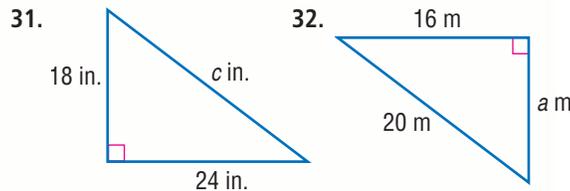
Example 4 Name all sets of numbers to which $-\sqrt{33}$ belongs.

$$-\sqrt{33} \approx -5.744562647$$

Since the decimal does not terminate or repeat, it is an irrational number.

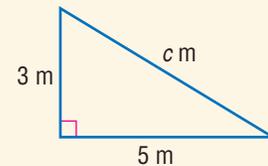
3-4 The Pythagorean Theorem (pp. 132–136)

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.



35. a , 5 in.; c , 6 in. 36. a , 6 cm; b , 7 cm

Example 5 Write an equation you could use to find the length of the hypotenuse of the right triangle. Then find the missing length.



$$c^2 = a^2 + b^2 \quad \text{Pythagorean Theorem}$$
$$c^2 = 3^2 + 5^2 \quad \text{Replace } a \text{ with 3 and } b \text{ with 5.}$$
$$c^2 = 9 + 25 \quad \text{Evaluate } 3^2 \text{ and } 5^2.$$
$$c^2 = 34 \quad \text{Simplify.}$$
$$c = \sqrt{34} \quad \text{Take the square root of each side.}$$
$$c \approx 5.8 \quad \text{Use a calculator.}$$

The hypotenuse is about 5.8 meters long.

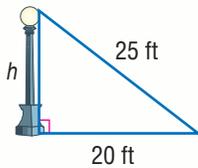
Mixed Problem Solving

For mixed problem-solving practice, see page 650.

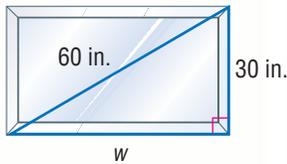
3-5 Using the Pythagorean Theorem (pp. 137–140)

Write an equation that can be used to answer the question. Then solve. Round to the nearest tenth if necessary.

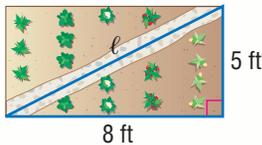
37. How tall is the light?



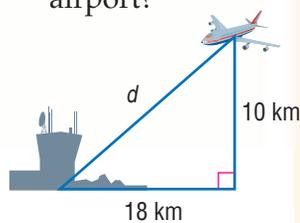
38. How wide is the window?



39. How long is the walkway?

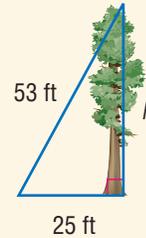


40. How far is the plane from the airport?



41. **GEOMETRY** A rectangle is 12 meters by 7 meters. What is the length of one of its diagonals?

- Example 6** Write an equation that can be used to find the height of the tree. Then solve.



Use the Pythagorean Theorem to write the equation $53^2 = h^2 + 25^2$. Then solve the equation.

$$53^2 = h^2 + 25^2 \quad \text{Evaluate } 53^2 \text{ and } 25^2.$$

$$2,809 = h^2 + 625$$

$$2,809 - 625 = h^2 + 625 - 625 \quad \text{Subtract } 625.$$

$$2,184 = h^2$$

$$\sqrt{2,184} = h \quad \text{Simplify.}$$

$$46.7 \approx h \quad \text{Take the square root of each side.}$$

$$46.7 \approx h \quad \text{Use a calculator.}$$

The height of the tree is about 47 feet.

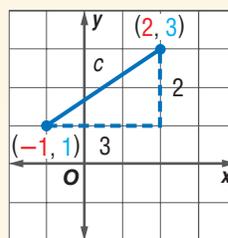
3-6 Geometry: Distance on the Coordinate Plane (pp. 142–145)

Graph each pair of ordered pairs. Then find the distance between the points. Round to the nearest tenth if necessary.

42. $(0, -3), (5, 5)$ 43. $(-1, 2), (4, 8)$
 44. $(-2, 1), (2, 3)$ 45. $(-6, 2), (-4, 5)$
 46. $(3, 4), (-2, 0)$ 47. $(-1, 3), (2, 4)$

48. **GEOMETRY** The coordinates of points R and S are $(4, 3)$ and $(1, 6)$. What is the distance between the points? Round to the nearest tenth if necessary.

- Example 7** Graph the ordered pairs $(2, 3)$ and $(-1, 1)$. Then find the distance between the points.



$$c^2 = a^2 + b^2$$

$$c^2 = 3^2 + 2^2$$

$$c^2 = 9 + 4$$

$$c^2 = 13$$

$$c = \sqrt{13}$$

$$c \approx 3.6$$

The distance is about 3.6 units.

Practice Test

Vocabulary and Concepts

- OPEN ENDED** Write an equation that can be solved by taking the square root of a perfect square.
- State the Pythagorean Theorem.

Skills and Applications

Find each square root.

3. $\sqrt{225}$

4. $-\sqrt{25}$

5. $\sqrt{\frac{36}{49}}$

Estimate to the nearest whole number.

6. $\sqrt{67}$

7. $\sqrt{108}$

8. $\sqrt{82}$

Name all sets of numbers to which each real number belongs.

9. $-\sqrt{64}$

10. $6.\overline{13}$

11. $\sqrt{14}$

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

12. $a, 5$ m; $b, 5$ m

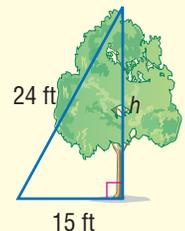
13. $b, 20$ ft; $c, 35$ ft

Determine whether each triangle with sides of given lengths is a right triangle.

14. 12 in., 20 in., 24 in.

15. 34 cm, 30 cm, 16 cm

16. **LANDSCAPING** To make a balanced landscaping plan for a yard, Kelsey needs to know the heights of various plants. How tall is the tree at the right?



17. **GEOMETRY** Find the perimeter of a right triangle with legs of 10 inches and 8 inches.

Graph each pair of ordered pairs. Then find the distance between points. Round to the nearest tenth if necessary.

18. $(-2, -2), (5, 6)$

19. $(1, 3), (-4, 5)$

Standardized Test Practice

20. **MULTIPLE CHOICE** If the area of a square is 40 square millimeters, what is the approximate length of one side of the square?

(A) 6.3 mm

(B) 7.5 mm

(C) 10 mm

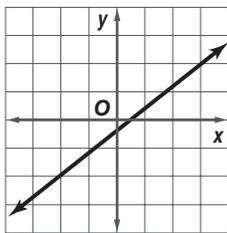
(D) 20 mm



PART 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Which of the following sets of ordered pairs represents two points on the line below? (Prerequisite Skill, p. 614)



- (A) $\{(3, 1), (2, -1)\}$ (B) $\{(3, 2), (-1, -2)\}$
 (C) $\{(3, 2), (-2, -2)\}$ (D) $\{(3, 3), (-2, -3)\}$

2. The table below shows the income of several baseball teams in a recent year. What is the total revenue for all of these teams? (Lesson 1-4)

Team	Income
Braves	-\$14,400,000
Orioles	\$1,500,000
Cubs	\$4,800,000
Tigers	\$500,000
Marlins	-\$27,700,000
Yankees	\$40,900,000
A's	-\$7,100,000
Pirates	-\$3,000,000

Source: www.mlb.com

- (F) \$99,900,000 (G) \$4,500,000
 (H) -\$4,500,000 (I) -\$99,900,000
3. Which of the following is equivalent to 0.64? (Lesson 2-1)
- (A) $\frac{1}{64}$ (B) $\frac{16}{25}$
 (C) $\frac{100}{64}$ (D) $\frac{64}{10}$

4. Which of the following values are equivalent? (Lesson 2-2)

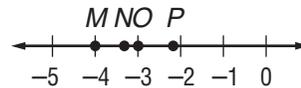
$0.08, 0.8, \frac{1}{8}, \frac{4}{5}$

- (F) 0.08 and $\frac{1}{8}$ (G) 0.8 and $\frac{1}{8}$
 (H) 0.08 and $\frac{4}{5}$ (I) 0.8 and $\frac{4}{5}$

5. Between which two whole numbers is $\sqrt{56}$ located on a number line? (Lesson 3-2)

- (A) 6 and 7 (B) 7 and 8
 (C) 8 and 9 (D) 9 and 10

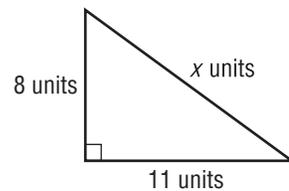
6. Which of the points on the number line is the best representation of $-\sqrt{11}$? (Lesson 3-3)



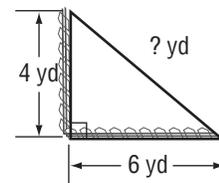
- (F) M (G) N (H) O (I) P

7. What is the value of x ? (Lesson 3-4)

- (A) $\sqrt{8 + 11}$
 (B) $\sqrt{8^2 + 11^2}$
 (C) $\frac{8^2 + 11^2}{2}$
 (D) $8^2 + 11^2$



8. Two fences meet in the corner of the yard. The length of one fence is 4 yards, and the other is 6 yards. What is the distance between the far ends of the fences? (Lesson 3-5)



- (F) 6.3 yd (G) 7.2 yd
 (H) 8.8 yd (I) 9.5 yd

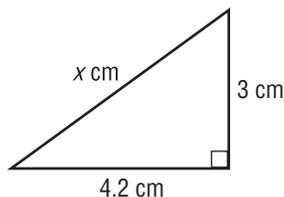
PART 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

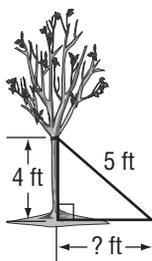
9. Missy placed a stick near the edge of the water on the beach. If the sum of the distances from the stick is positive, the tide is coming in. If the sum of the distances is negative, the tide is going out. Determine whether the tide is coming in or going out for the readings at the right. (Lesson 1-4)
10. Is the square root of 25 equal to 5, -5 , or both? (Lesson 3-1)
11. The value of $\sqrt{134}$ is between what two consecutive whole numbers? (Lesson 3-2)

Wave Distance from Stick (inches)
+3
+5
-4
+2
+8
-6
-3
+7
-5
-4

12. Find the value of x to the nearest tenth. (Lesson 3-4)



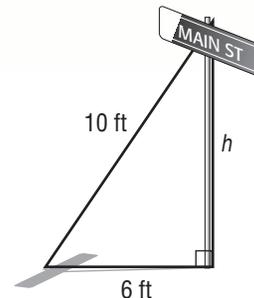
13. Lucas attaches a wire to a young oak tree 4 feet above the ground. The wire is anchored in the ground at an angle from the tree to help the tree stay upright as it grows. If the wire is 5 feet long, what is the distance from the base of the wire to the base of the tree? (Lesson 3-4)



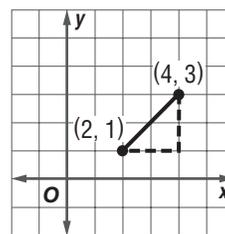
TEST-TAKING TIP

Questions 12 and 13 Remember that the hypotenuse of a right triangle is always opposite the right angle.

14. A signpost casts a shadow that is 6 feet long. The top of the post is 10 feet from the end of the shadow. What is the height of the post? (Lesson 3-5)



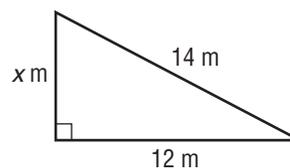
15. Find the distance between the points located on the graph below. Round to the nearest tenth. (Lesson 3-6)



PART 3 Extended Response

Record your answers on a sheet of paper. Show your work.

16. Use the right triangle to answer the following questions. (Lesson 3-4)



- Write an equation that can be used to find the length of x .
 - Solve the equation. Justify each step.
 - What is the length of x ?
17. Use a grid to graph and answer the following questions. (Lesson 3-6)
- Graph the ordered pairs $(3, 4)$ and $(-2, 1)$.
 - Describe how to find the distance between the two points.
 - Find the distance between the points.

