Scientists place radio frequency tags on some animals within a population of that species. Then they track data, such as migration patterns, about the animals.
Complete these exercises to review skills you will need for this module.

### Fractions, Decimals, and Percents

**EXAMPLE**

Write \(\frac{13}{20}\) as a decimal and a percent.

\[
\begin{align*}
\text{decimal} & : \quad 20 \div 13.00 = 0.65 \\
\text{percent} & : \quad 0.65 = 65\%
\end{align*}
\]

Write each fraction as a decimal and a percent.

1. \(\frac{7}{8}\) \hspace{1cm} 2. \(\frac{4}{5}\) \hspace{1cm} 3. \(\frac{1}{4}\) \hspace{1cm} 4. \(\frac{3}{10}\)

### Find the Median and Mode

**EXAMPLE**

Order the data from least to greatest.

\[
\text{median} = \frac{13 + 14}{2} = 13.5
\]

The mode is the item that appears most frequently in the data.

Find the median and the mode of the data.

5. \(11, 17, 7, 6, 7, 4, 15, 9\) \hspace{1cm} 6. \(43, 37, 49, 51, 56, 40, 44, 50, 36\)

### Find the Mean

**EXAMPLE**

The mean is the sum of the data items divided by the number of items.

\[
\begin{align*}
\text{mean} & : \quad \frac{17 + 14 + 13 + 16 + 13 + 11}{6} = \frac{84}{6} = 14
\end{align*}
\]

Find the mean of the data.

7. \(9, 16, 13, 14, 10, 16, 17, 9\) \hspace{1cm} 8. \(108, 95, 104, 96, 97, 106, 94\)
Active Reading

Layered Book  Before beginning the module, create a layered book to help you learn the concepts in this module. Label the first flap with the module title. Label the remaining flaps with the lesson titles. As you study each lesson, write important ideas, such as vocabulary and formulas, under the appropriate flap. Refer to your finished layered book as you work on exercises from this module.
WHAT IT MEANS TO YOU

You will compare two populations based on random samples.

EXAMPLE 7.SP.3

Melinda surveys a random sample of 16 students from two college dorms to find the average number of hours of sleep they get. Use the results shown in the dot plots to compare the two populations.

**Average Daily Hours of Sleep**

Students in Jones Hall tend to sleep more than students in Anderson Hall, but the variation in the data sets is similar.

WHAT IT MEANS TO YOU

You will compare two groups of data by comparing the difference in the means to the variability.

EXAMPLE 7.SP.3

The tables show the number of items that students in a class answered correctly on two different math tests. How does the difference in the means of the data sets compare to the variability?

<table>
<thead>
<tr>
<th>Items Correct on Test 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>20, 13, 18, 19, 15, 18, 20, 20, 15, 15, 19, 18</td>
</tr>
<tr>
<td>Mean: 17.5; Mean absolute deviation: 2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Items Correct on Test 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>8, 12, 12, 8, 15, 16, 14, 12, 13, 9, 14, 11</td>
</tr>
<tr>
<td>Mean: 12; Mean absolute deviation: 2</td>
</tr>
</tbody>
</table>

The means of the two data sets differ by $\frac{17.5 - 12}{2} = 2.75$ times the variability of the data sets.
EXPLORE ACTIVITY

Analyzing Dot Plots

You can use dot plots to analyze a data set, especially with respect to its center and spread.

People once used body parts for measurements. For example, an inch was the width of a man’s thumb. In the 12th century, King Henry I of England stated that a yard was the distance from his nose to his outstretched arm’s thumb. The dot plot shows the different lengths, in inches, of the “yards” for students in a 7th grade class.

A. Describe the shape of the dot plot. Are the dots evenly distributed or grouped on one side?

B. Describe the center of the dot plot. What single dot would best represent the data?

C. Describe the spread of the dot plot. Are there any outliers?

Reflect

1. Calculate the mean, median, and range of the data in the dot plot.
Comparing Dot Plots Visually
You can compare dot plots visually using various characteristics, such as center, spread, and shape.

EXAMPLE 1

The dot plots show the heights of 15 high school basketball players and the heights of 15 high school softball players.

A Visually compare the shapes of the dot plots.
Softball: All the data is 5’6” or less.
Basketball: Most of the data is 5’8” or greater.
As a group, the softball players are shorter than the basketball players.

B Visually compare the centers of the dot plots.
Softball: The data is centered around 5’4”.
Basketball: The data is centered around 5’8”.
This means that the most common height for the softball players is 5 feet 4 inches, and for the basketball players 5 feet 8 inches.

C Visually compare the spreads of the dot plots.
Softball: The spread is from 4’11” to 5’6”.
Basketball: The spread is from 5’2” to 6’0”.
There is a greater spread in heights for the basketball players.

YOUR TURN

2. Visually compare the dot plot of heights of field hockey players to the dot plots for softball and basketball players.

Shape: ____________________________________________

Center: ____________________________________________

Spread: ____________________________________________
Comparing Dot Plots Numerically

You can also compare the shape, center, and spread of two dot plots numerically by calculating values related to the center and spread. Remember that outliers can affect your calculations.

**EXAMPLE 2**

Numerically compare the dot plots of the number of hours a class of students exercises each week to the number of hours the students play video games each week.

A. Compare the shapes of the dot plots.

The dot plots appear almost opposite. The dot plots show that most students **exercise** less than 4 hours but most play **video games** more than 6 hours each week.

B. Compare the centers of the dot plots by finding the medians.

The median number of hours that students **exercise** is 2.5 hours, which is 6.5 hours less than the median of 9 hours that students play **video games**.

C. Compare the spreads of the dot plots by calculating the ranges.

The dot plots show the ranges to be similar to one another. The range for the amount of time that students **exercise** is 12 hours, and the range for the amount of time that students play **video games** is 14 hours.

**YOUR TURN**

3. Calculate the median and range of the data in the dot plot. Then compare the results to the dot plot for exercise in Example 2.

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________
The dot plots show the number of miles run per week for two different classes. For 1–5, use the dot plots shown. (Explore Activity, Example 1 and 2)

1. Compare the shapes of the dot plots.

2. Compare the centers of the dot plots.

3. Compare the spreads of the dot plots.

4. Calculate the medians of the dot plots.

5. Calculate the ranges of the dot plots.

6. What do the medians and ranges of two dot plots tell you about the data?
The dot plot shows the number of letters in the spellings of the 12 months. Use the dot plot for 7–10.

7. Describe the shape of the dot plot.

8. Describe the center of the dot plot.

9. Describe the spread of the dot plot.

10. Calculate the mean, median, and range of the data in the dot plot.

The dot plots show the mean number of days with rain per month for two cities.

11. Compare the shapes of the dot plots.

12. Compare the centers of the dot plots.

13. Compare the spreads of the dot plots.

14. What do the dot plots tell you about the two cities with respect to their average monthly rainfall?
The dot plots show the shoe sizes of two different groups of people.

15. Compare the shapes of the dot plots.

16. Compare the medians of the dot plots.

17. Compare the ranges of the dot plots (with and without the outliers).

18. Make A Conjecture  Provide a possible explanation for the results of the dot plots.

19. Analyze Relationships  Can two dot plots have the same median and range but have completely different shapes? Justify your answer using examples.

20. Draw Conclusions  What value is most affected by an outlier, the median or the range? Explain. Can you see these effects in a dot plot?
How do you compare two sets of data displayed in box plots?

**LESSON 10.2 Comparing Data Displayed in Box Plots**

**ESSENTIAL QUESTION**
How do you compare two sets of data displayed in box plots?

**EXPLORATION ACTIVITY**

**Analyzing Box Plots**

Box plots show five key values to represent a set of data, the least and greatest values, the lower and upper quartile, and the median. To create a box plot, arrange the data in order, and divide them into four equal-size parts or quarters. Then draw the box and the whiskers as shown.

The number of points a high school basketball player scored during the games he played this season are organized in the box plot shown.

![Box Plot]

### A
Find the least and greatest values.

Least value: _____________  Greatest value: _____________

### B
Find the median and describe what it means for the data.

________________________________________________________________________

________________________________________________________________________

### C
Find and describe the lower and upper quartiles.

________________________________________________________________________

________________________________________________________________________

### D
The interquartile range is the difference between the lower and upper quartiles, which is represented by the length of the box. Find the interquartile range.

\[ Q_3 - Q_1 = \] _____________ - _____________ = _____________

**Math Talk**

How do the lengths of the whiskers compare? Explain what this means.

**Mathematical Practices**

7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. Also, 7.SP.4
Reflect
1. Why is one-half of the box wider than the other half of the box?

Box Plots with Similar Variability
You can compare two box plots numerically according to their centers, or medians, and their spreads, or variability. Range and interquartile range (IQR) are both measures of spread. Box plots with similar variability should have similar boxes and whiskers.

**EXAMPLE 1**

The box plots show the distribution of times spent shopping by two different groups.

A Compare the shapes of the box plots.
The positions and lengths of the boxes and whiskers appear to be very similar. In both plots, the right whisker is shorter than the left whisker.

B Compare the centers of the box plots.
Group A's median, 47.5, is greater than Group B's, 40. This means that the median shopping time for Group A is 7.5 minutes more.

C Compare the spreads of the box plots.
The box shows the interquartile range. The boxes are similar.

Group A: $55 - 30 = 25$ min  
Group B: About $59 - 32 = 26$ min

The whiskers have similar lengths, with Group A's slightly shorter than Group B's.

Reflect
2. Which group has the greater variability in the bottom 50% of shopping times? The top 50% of shopping times? Explain how you know.
Box Plots with Different Variability

You can compare box plots with greater variability, where there is less overlap of the median and interquartile range.

EXAMPLE 2

The box plots show the distribution of the number of team wristbands sold daily by two different stores over the same time period.

A. Compare the shapes of the box plots.
   Store A’s box and right whisker are longer than Store B’s.

B. Compare the centers of the box plots.
   Store A’s median is about 43, and Store B’s is about 51. Store A’s median is close to Store B’s minimum value, so about 50% of Store A’s daily sales were less than sales on Store B’s worst day.

C. Compare the spreads of the box plots.
   Store A has a greater spread. Its range and interquartile range are both greater. Four of Store B’s key values are greater than Store A’s corresponding value. Store B had a greater number of sales overall.
For 1–3, use the box plot Terrence created for his math test scores. Find each value. (Explore Activity)

1. Minimum = _______ Maximum = _______
2. Median = _______
3. Range = _______ IQR = _______

For 4–7, use the box plots showing the distribution of the heights of hockey and volleyball players. (Examples 1 and 2)

4. Which group has a greater median height? __________________________
5. Which group has the shortest player? __________________________
6. Which group has an interquartile range of about 10? __________________________

7. What information can you use to compare two box plots?
   ____________________________________________
For 8–11, use the box plots of the distances traveled by two toy cars that were jumped from a ramp.

8. Compare the minimum, maximum, and median of the box plots.

9. Compare the ranges and interquartile ranges of the data in box plots.

10. What do the box plots tell you about the jump distances of two cars?

11. Critical Thinking What do the whiskers tell you about the two data sets?

For 12–14, use the box plots to compare the costs of leasing cars in two different cities.

12. In which city could you spend the least amount of money to lease a car? The greatest?

13. Which city has a higher median price? How much higher is it?

14. Make a Conjecture In which city is it more likely to choose a car at random that leases for less than $450? Why?
15. **Summarize** Look back at the box plots for 12–14 on the previous page. What do the box plots tell you about the costs of leasing cars in those two cities?

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

16. **Draw Conclusions** Two box plots have the same median and equally long whiskers. If one box plot has a longer box than the other box plot, what does this tell you about the difference between the data sets?

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

17. **Communicate Mathematical Ideas** What can you learn about a data set from a box plot? How is this information different from a dot plot?

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

18. **Analyze Relationships** In mathematics, *central tendency* is the tendency of data values to cluster around some central value. What does a measure of variability tell you about the central tendency of a set of data? Explain.

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________
ESSENTIAL QUESTION
How can you use statistical measures to compare populations?

Comparing Differences in Centers to Variability
Recall that to find the mean absolute deviation (MAD) of a data set, first find the mean of the data. Next, take the absolute value of the difference between the mean and each data point. Finally, find the mean of those absolute values.

EXAMPLE 1
The tables show the number of minutes per day students in a class spend exercising and playing video games. What is the difference of the means as a multiple of the mean absolute deviations?

<table>
<thead>
<tr>
<th>Minutes Per Day Exercising</th>
</tr>
</thead>
<tbody>
<tr>
<td>0, 7, 7, 18, 20, 38, 33, 24, 22, 18, 11, 6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Minutes Per Day Playing Video Games</th>
</tr>
</thead>
<tbody>
<tr>
<td>13, 18, 19, 30, 32, 46, 50, 34, 36, 30, 23, 19</td>
</tr>
</tbody>
</table>

STEP 1
Calculate the mean number of minutes per day exercising.

\[0 + 7 + 7 + 18 + 20 + 38 + 33 + 24 + 22 + 18 + 11 + 6 = 204\]

\[204 \div 12 = 17\] Divide the sum by the number of students.

STEP 2
Calculate the mean absolute deviation for the number of minutes exercising.

\[|0 - 17| = 17, \quad |7 - 17| = 10, \quad |7 - 17| = 10, \quad |18 - 17| = 1\]

\[|20 - 17| = 3, \quad |38 - 17| = 21, \quad |33 - 17| = 16, \quad |24 - 17| = 7\]

\[|22 - 17| = 5, \quad |18 - 17| = 1, \quad |11 - 17| = 6, \quad |6 - 17| = 11\]

Find the mean of the absolute values.

\[17 + 10 + 10 + 1 + 3 + 21 + 16 + 7 + 5 + 1 + 6 + 11 = 108\]

\[108 \div 12 = 9\] Divide the sum by the number of students.
**STEP 3** Calculate the mean number of minutes per day playing video games. Round to the nearest tenth.

\[
13 + 18 + 19 + 30 + 32 + 46 + 50 + 34 + 36 + 30 + 23 + 19 = 350
\]

\[
350 \div 12 \approx 29.2 \quad \text{Divide the sum by the number of students.}
\]

**STEP 4** Calculate the mean absolute deviation for the numbers of minutes playing video games.

\[
|13 - 29.2| = 16.2 \quad |18 - 29.2| = 11.2 \quad |19 - 29.2| = 10.2
\]

\[
|30 - 29.2| = 0.8 \quad |32 - 29.2| = 2.8 \quad |46 - 29.2| = 16.8
\]

\[
|50 - 29.2| = 20.8 \quad |34 - 29.2| = 4.8 \quad |36 - 29.2| = 6.8
\]

\[
|30 - 29.2| = 0.8 \quad |23 - 29.2| = 6.2 \quad |19 - 29.2| = 10.2
\]

Find the mean of the absolute values. Round to the nearest tenth.

\[
16.2 + 11.2 + 10.2 + 0.8 + 2.8 + 16.8 + 20.8 + 4.8 + 6.8 + 0.8 + 6.2 + 10.2 = 107.6
\]

\[
107.6 \div 12 \approx 9 \quad \text{Divide the sum by the number of students.}
\]

**STEP 5** Find the difference in the means.

\[
29.2 - 17 = 12.2 \quad \text{Subtract the lesser mean from the greater mean.}
\]

**STEP 6** Write the difference of the means as a multiple of the mean absolute deviations, which are similar but not identical.

\[
12.2 \div 9 \approx 1.36 \quad \text{Divide the difference of the means by the MAD.}
\]

The means of the two data sets differ by about 1.4 times the variability of the two data sets.

---

**YOUR TURN**

1. The high jumps in inches of the students on two intramural track and field teams are shown below. What is the difference of the means as a multiple of the mean absolute deviations?

<table>
<thead>
<tr>
<th>High Jumps for Students on Team 1 (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>44, 47, 67, 89, 55, 76, 85, 80, 87, 69, 47, 58</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>High Jumps for Students on Team 2 (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40, 32, 52, 75, 65, 70, 72, 61, 54, 43, 29, 32</td>
</tr>
</tbody>
</table>
Using Multiple Samples to Compare Populations

Many different random samples are possible for any given population, and their measures of center can vary. Using multiple samples can give us an idea of how reliable any inferences or predictions we make are.

EXAMPLE 2

A group of about 250 students in grade 7 and about 250 students in grade 11 were asked, “How many hours per month do you volunteer?” Responses from one random sample of 10 students in grade 7 and one random sample of 10 students in grade 11 are summarized in the box plots.

Two Random Samples of Size 10

Grade 7

Grade 11

How can we tell if the grade 11 students do more volunteer work than the grade 7 students?

STEP 1

The median is higher for the students in grade 11. But there is a great deal of variation. To make an inference for the entire population, it is helpful to consider how the medians vary among multiple samples.

STEP 2

The box plots below show how the medians from 10 different random samples for each group vary.

Distribution of Medians from 10 Random Samples of Size 10

Grade 7

Grade 11

The medians vary less than the actual data. Half of the grade 7 medians are within 1 hour of 9. Half of the grade 11 medians are within 1 or 2 hours of 11. Although the distributions overlap, the middle halves of the data barely overlap. This is fairly convincing evidence that the grade 11 students volunteer more than the grade 7 students.
Guided Practice

The tables show the numbers of miles run by the students in two classes. Use the tables in 1–2. (Example 1)

<table>
<thead>
<tr>
<th>Miles Run by Class 1 Students</th>
<th>Miles Run by Class 2 Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>12, 1, 6, 10, 1, 2, 3, 10, 3, 9, 8, 6, 8</td>
<td>11, 14, 11, 13, 6, 7, 8, 6, 8, 13, 8, 15, 13, 17, 15</td>
</tr>
</tbody>
</table>

1. For each class, what is the mean? What is the mean absolute deviation (MAD)?

2. The difference of the means is about _________ times the (MAD).

3. The box plots show the distributions of mean weights of 10 samples of 10 football players from each of two leagues, A and B. What can you say about any comparison of the weights of the two populations? Explain.

4. Why is it a good idea to use multiple random samples when making comparative inferences about two populations?
Josie recorded the average monthly temperatures for two cities in the state where she lives. Use the data for 5–7.

Average Monthly Temperatures for City 1 (°F)
23, 38, 39, 48, 55, 56, 71, 86, 57, 53, 43, 31

Average Monthly Temperatures for City 2 (°F)
8, 23, 24, 33, 40, 41, 56, 71, 42, 38, 28, 16

5. For City 1, what is the mean of the average monthly temperatures? What is the mean absolute deviation of the average monthly temperatures?

6. What is the difference between each average monthly temperature for City 1 and the corresponding temperature for City 2? ______________

7. Draw Conclusions Based on your answers to Exercises 5 and 6, what do you think the mean of the average monthly temperatures for City 2 is? What do you think the mean absolute deviation of the average monthly temperatures for City 2 is? Give your answers without actually calculating the mean and the mean absolute deviation. Explain your reasoning. ______________

8. What is the difference in the means as a multiple of the mean absolute deviations? ______________

9. Make a Conjecture Mark took 10 random samples of 10 students from two schools. He asked how many minutes they spend per day going to and from school. The tables show the medians and the means of the samples. Compare the travel times using distributions of the medians and means.

<table>
<thead>
<tr>
<th>School A</th>
<th>School B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medians: 28, 22, 25, 10, 40, 36, 30, 14, 20, 25</td>
<td>Medians: 22, 25, 20, 14, 20, 18, 21, 18, 26, 19</td>
</tr>
<tr>
<td>Means: 27, 24, 27, 15, 42, 36, 32, 18, 22, 29</td>
<td>Means: 24, 30, 22, 15, 20, 17, 22, 15, 36, 27</td>
</tr>
</tbody>
</table>
10. **Justify Reasoning** Statistical measures are shown for the ages of middle school and high school teachers in two states.

**State A:** Mean age of middle school teachers = 38, mean age of high school teachers = 48, mean absolute deviation for both = 6

**State B:** Mean age of middle school teachers = 42, mean age of high school teachers = 50, mean absolute deviation for both = 4

In which state is the difference in ages between members of the two groups more significant? Support your answer.

11. **Analyze Relationships** The tables show the heights in inches of all the adult grandchildren of two sets of grandparents, the Smiths and the Thompsons. What is the difference in the medians as a multiple of the ranges?

<table>
<thead>
<tr>
<th>Heights of the Smiths’ Adult Grandchildren (in.)</th>
<th>Heights of the Thompsons’ Adult Grandchildren (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>64, 65, 68, 66, 65, 68, 69, 66, 70, 67</td>
<td>75, 80, 78, 77, 79, 76, 75, 79, 77, 74</td>
</tr>
</tbody>
</table>

12. **Critical Thinking** Jill took many samples of 10 tosses of a standard number cube. What might she reasonably expect the median of the medians of the samples to be? Why?

13. **Analyze Relationships** Elly and Ramon are both conducting surveys to compare the average numbers of hours per month that men and women spend shopping. Elly plans to take many samples of size 10 from both populations and compare the distributions of both the medians and the means. Ramon will do the same, but will use a sample size of 100. Whose results will probably produce more reliable inferences? Explain.

14. **Counterexamples** Seth believes that it is always possible to compare two populations of numerical values by finding the difference in the means of the populations as a multiple of the mean absolute deviations. Describe a situation that explains why Seth is incorrect.
10.1 Comparing Data Displayed in Dot Plots

The two dot plots show the number of miles run by 14 students at the start and at the end of the school year. Compare each measure for the two dot plots. Use the data for 1–3.

1. means __________________________
2. medians _________________________
3. ranges __________________________

10.2 Comparing Data Displayed in Box Plots

The box plots show lengths of flights in inches flown by two model airplanes. Use the data for 4–5.

4. Which has a greater median flight length? __________
5. Which has a greater interquartile range? __________

10.3 Using Statistical Measures to Compare Populations

6. Roberta grows pea plants, some in shade and some in sun. She picks 8 plants of each type at random and records the heights.

<table>
<thead>
<tr>
<th>Shade plant heights (in.)</th>
<th>7 11 11 12 9 12 8 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun plant heights (in.)</td>
<td>21 24 19 19 22 23 24 24</td>
</tr>
</tbody>
</table>

Express the difference in the means as a multiple of their ranges.

7. How can you use and compare data to solve real-world problems?

__________________________________________
1. The box plots show the price of a gallon of gasoline in two cities. Look at each measure. Is the measure greater for city B?
Select Yes or No for A–C.
A. median  ○ Yes  ○ No
B. range  ○ Yes  ○ No
C. interquartile range  ○ Yes  ○ No

2. Gabrielle surveyed a random sample of students about the amount of time they spent on math and science homework yesterday. Her results are shown in the dot plots.
Choose True or False for each statement.
A. Neither data set has an outlier.  ○ True  ○ False
B. The science data is more symmetric.  ○ True  ○ False
C. The science data has a greater mean.  ○ True  ○ False

3. Use the data from Item 2. What is the difference of the means as a multiple of the mean absolute deviations? Explain how you solved this problem.

4. Tyrone buys a pair of shoes on sale for 20% off. The regular price of the shoes is $59.95, and the sales tax rate is 7.5%. How much will Tyrone pay for the shoes, including sales tax? Explain how you solved this problem.